GESC18 Two- and Three-body YNN Potentials, $\Lambda N, \Sigma N, \Xi N$ G-matrix Application

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Abstract

Effective Hyperon-nucleon Two-Meson-Pair Exchange potentials are derived for the $(\pi\rho)_1$-pairs etc. that figure in the Extended-soft-core (ESC) models for Baryon-baryon scattering [1, 2]. For comparison we also give, in an appendix, the Fujita-Miyazawa three-nucleon potentials.

The pair-vertices contain: (i) heavy-boson effects, (ii) resonance effects, and (iii) negative-energy baryon contributions. The first two effects have been discussed in [3, 4]. Item (iii) see [5, 6], where it is shown that the negative energy intermediate state baryon contributions can be described by an effective interaction Hamiltonian. The two-body integral equations for positive-energy baryons becomes relativistic covariant.

The out-integration of the "third" nucleon reduces the three-body contribution, and only from the pair-interactions included in ESC04, ESC08 the contributions from $(\pi\pi)_0, (\pi\eta), (\pi\sigma)$ and $(\pi\omega)$ pairs survive. The G-matrix model GESC18 includes the multi-pomeron (MPP) repulsion, and we added the meson-pair vertices $(\sigma\sigma), (VV)$ and (AA).

The results of the inclusion of the three-body interactions generated by the pair-interactions are: (a) The $(\pi\eta), (\pi\sigma), (\pi\omega)$ pair interactions lead to $U_{\sigma\sigma} \geq +1.5$; (b) The Λ-nucleus interaction is strong enough to support the Isaka-Yamamoto calculations on Λ-hypernuclei; (c) The $(\sigma\sigma)$ pair interactions give the possibility of a sizeable attraction in $\Xi N(^3S_1, T=1)$; (d) The MPP-strength can be taken as large as to support the $2M_\odot$ neutron stars.

PACS numbers:

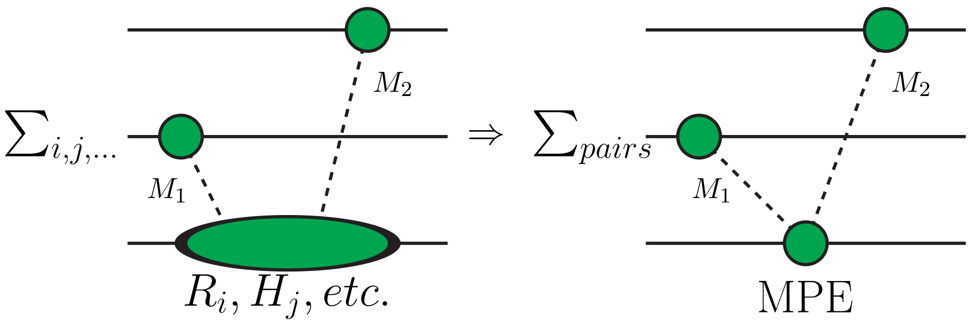

FIG. 1: *Meson-pair description and low-energy approximation.*

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I. INTRODUCTION

Meson-Pair-Exchange (MPE) YNN potentials are derived for all meson-pairs that figure in the Extended-soft-core (ESC) models for Baryon-baryon scattering [1, 2].
Two-meson-exchange three-body-potentials have been studied already for a long time, starting with Primakov and Holstein [7], and followed by Drell and Huang [8], Miyazawa and Fujita [9, 10]. The Tucson-Melbourne three-body force [11, 12] is based on extrapolations of the pion-nucleon amplitudes, phenomenological input, and using PCAC and current algebra for controlling the form of the (off-shell) pion-nucleon momenta, with an emphasis on (broken) chiral-symmetry (BCS). However, these constructions are not done in close correspondence with a *realistic* two-body force. Later Grangè *et al* [13] worked out a rather complete three-nucleon potential for the Paris potentials [14]. Here, important contributions were alluded to nucleon-antinucleon ($N\bar{N}$) pairs contributions.

It is the aim of this paper to derive the three-nucleon potential corresponding to the dynamics contained in the ESC-models. Very early, pion-pair exchange in connection with the three-nucleon potential was discussed and in principle consistent with the two-nucleon force [8]. Especially, with the ESC-model, we are in an ideal position to derive three-body BB-potentials, which are consistent with the two-body BB-potentials, because it is the only realistic baryon-baryon interaction model that incorporates meson-pair-exchange as well as one-boson-exchange (OBE). All the occurring parameters were fitted to the Nijmegen partial wave analysis [15] for the two-nucleon scattering data for $T_{lab} \leq 350$ MeV, and the low-energy data, reaching $\chi^2_{p.d.p.} = 1.09$, see for details [16].

Now, because of the large cancellations that take place from the iterated OBE-potentials in the three-body systems, a particular prominent role is played by the meson-pair interactions. This was recognized already a long time agoo. In the fifties and sixties three-body potentials were worked out already [8, 17], and also the effect of excited intermediate states involving e.g. the Δ_{33}-resonance [9]. Since all the effects of the heavy-bosons and meson-baryon resonances are implicitly included in the pair-interactions of the ESC-model, a rather complete description of the three-body forces will emerge by working out all the OBE-iterated, and mesoson-pair-exchanges (MPE).

It has been shown in [18] that the three-body potentials from the iterated OBE-exchanges vanish in the absence of negative-energy nucleon-sates are absolutely suppressed, which we assume in the ESC-model approach. Then, this brings forward the prominence of the meson-pair contributions to the three-body force.

The methods used in this treatise on the three-nucleon force were developed in [19] and applied in [3, 4].

The content of these notes is as follows: In section II the meson pair interactions Hamiltonians are defined, the BB matrix elements in Dirac and Pauli spinor space, and the YNN-graphs are given. In section III the Feynman diagram calculation is worked out, taking the $(\pi\rho)_!$-pair as an example. In section IV the three-particle potential in configuration space is described. In section V the two-meson-pair exchange three-body potentials are listed for all pair types. In section VI the effective two-body potentials in nuclear matter are derived by the out-integration of the "third" nucleon. In section VII the LNR-approximation is applied, which leads to rather drastic reduction in the contributions. In section VIII the configuration-space three-body induced hyperon-nucleon potentials are derived. In section IX the multi-pomeron effective 2-body potentials are described. Sections X, XI the $\Lambda N, \Sigma N$, and ΞN G-matrix applications. Section XII gives the result for the nuclear saturation. Sections XIII and XIV contain the Conclusions, Outlook, and Discussion.

The notes contain a number of both technical and physics appendices: In appendix A the eaxact reduction of Dirac-spinors to Pauli-spinors is given. Appendix D describes the Fourier transformation for non-local potentials. Appendices E, F, and G contain differentiation formules, Fourier integrals, and the transformation to configuration space. Appendix I gives the SU(3) structure of the pair-couplings, and the meson mixing. Appendix J discusses the SU(3) structure of the "effective" potentials. Appendix H treats the Fujita-Miyazawa potential.

Appendix L and M derive the σVV- and σAA-potentials respectively. Appendix N gives an estimate of the $\sigma\sigma\sigma$-coupling based on local chiral symmetry and mean-field (MF) models. In appendix O the Pomeron contributions to the $(\sigma\sigma), (VV)_0$ and $(AA)_0$ pair couplings is estimated. Finally, in Appendix N the two-body potentials due to the $(\sigma\sigma)_0, (VV)_0$, and $(AA)_0$ pairs are derived and discussed.

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II. THE MESON-PAIR INTERACTIONS

Before we can proceed and calculate the pair-meson potentials, we have to define the nucleon-nucleon-meson (NNm) and nucleon-nucleon-meson-meson (NNm_1m_2) Hamiltonians. For point couplings the nucleon-nucleon-meson Hamiltonians are [20]

$$\begin{aligned}
\mathcal{H}_{PV} &= \frac{f_P}{m_\pi}\bar{\psi}\gamma_5\gamma_\mu\boldsymbol{\tau}\psi\cdot\partial^\mu\boldsymbol{\phi}_P, && \text{(2.1a)}\\
\mathcal{H}_V &= g_V\bar{\psi}\gamma_\mu\boldsymbol{\tau}\psi\cdot\boldsymbol{\phi}_V^\mu - \frac{f_V}{2M}\bar{\psi}\sigma_{\mu\nu}\boldsymbol{\tau}\psi\cdot\partial^\nu\boldsymbol{\phi}_V^\mu\ , && \text{(2.1b)}\\
\mathcal{H}_S &= g_S\bar{\psi}\boldsymbol{\tau}\psi\cdot\boldsymbol{\phi}_S\ , && \text{(2.1c)}
\end{aligned}$$

where $\boldsymbol{\phi}$ denotes the pseudovector-, vector-, and scalar-meson field, respectively. For the isospin $I = 0$ mesons, the isospin Pauli matrices, $\boldsymbol{\tau}$, are absent.

A. Meson-Pair Interaction Hamiltonians

For the phenomenological meson-pair interactions the Hamiltonians, for meson-pairs with quantum numbers (J,P,C), are

$$
\begin{aligned}
J^{PC}=0^{++}:\ \mathcal{H}_S &= \bar{\psi}\psi\left[g^{(1)}_{(\pi\pi)_0}\boldsymbol{\pi}\cdot\boldsymbol{\pi}+g^{(2)}_{(\pi\pi)_0}\partial^\mu\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi}/m_\pi^2+g_{(\sigma\sigma)}\sigma^2\right]/m_\pi, && (2.2a)\\
\mathcal{H}_E &= \bar{\psi}\boldsymbol{\tau}\psi\cdot\boldsymbol{\pi}\left[g_{(\pi\eta)}\eta+g_{(\pi\eta')}\eta'\right]/m_\pi, && (2.2b)\\
J^{PC}=1^{--}:\ \mathcal{H}_V &= g_{(\pi\pi)_1}\bar{\psi}\gamma_\mu\boldsymbol{\tau}\psi\cdot(\boldsymbol{\pi}\times\partial^\mu\boldsymbol{\pi})/m_\pi^2\\
&\quad -\frac{f_{(\pi\pi)_1}}{2M}\bar{\psi}\sigma_{\mu\nu}\boldsymbol{\tau}\psi\partial^\nu\cdot(\boldsymbol{\pi}\times\partial^\mu\boldsymbol{\pi})/m_\pi^2, && (2.2c)\\
J^{PC}=1^{++}:\ \mathcal{H}_A &= g_{(\pi\rho)_1}\bar{\psi}\gamma_5\gamma_\mu\boldsymbol{\tau}\psi\cdot(\boldsymbol{\pi}\times\boldsymbol{\rho}^\mu)/m_\pi, && (2.2d)\\
\mathcal{H}_P &= g_{(\pi\sigma)}\bar{\psi}\gamma_5\gamma_\mu\boldsymbol{\tau}\psi\cdot(\boldsymbol{\pi}\partial^\mu\sigma-\sigma\partial^\mu\boldsymbol{\pi})/m_\pi^2\\
&\quad + g_{(\pi P)}\bar{\psi}\gamma_5\gamma_\mu\boldsymbol{\tau}\psi\cdot(\boldsymbol{\pi}\partial^\mu P-P\partial^\mu\boldsymbol{\pi})/m_\pi^2, && (2.2e)\\
J^{PC}=1^{+-}:\ \mathcal{H}_H &= -ig_{(\pi\rho)_0}\bar{\psi}\gamma_5\sigma_{\mu\nu}\psi\partial^\nu(\boldsymbol{\pi}\cdot\boldsymbol{\rho}^\mu)/m_\pi^2, && (2.2f)\\
\mathcal{H}_B &= -ig_{(\pi\omega)}\bar{\psi}\gamma_5\sigma_{\mu\nu}\boldsymbol{\tau}\psi\cdot\partial^\nu(\boldsymbol{\pi}\,\omega^\mu)/m_\pi^2. && (2.2g)
\end{aligned}
$$

In Eq. (2.2e) we have included the Pomeron contribution, but in recent ESC-models $g_{(\pi P)}=0$.

As for the scaling of the pair-coupling parameters, we have choosen the π^+-marss. For the operators $\partial^\mu\pi(x)$ this follows the non-linear chiral models. The other scaling m_π-factors may be could be better replaced by M, the nucleon mass. This would presumably represent better the scale of the physics involved. For example pair-couplings from $N\bar{N}$-pairs ('negative-energy staes') would be parameterized more naturally this way. However, in our works on the ESC-model we sofar always used the m_π-mass as a scaling parameter, and therefore we will do this also in this paper.

The transition from Dirac spinors to Pauli spinors is reviewed in Appendix C of [19]. Following this reference and keeping only terms up to order $1/M$, we find that the vertex operators in Pauli-spinor space for the NNm vertices are given by

$$\bar{u}(\mathbf{p}')\Gamma^{(1)}_P u(\mathbf{p}) = -i\frac{f_P}{m_\pi}\left[\boldsymbol{\sigma}_1\cdot\mathbf{k} \pm \frac{\omega}{2M}\boldsymbol{\sigma}_1\cdot(\mathbf{p}'+\mathbf{p})\right], \tag{2.3a}$$

$$\bar{u}(\mathbf{p}')\Gamma^{(1)}_V u(\mathbf{p}) = g_V\left[\left\{\left(1+\frac{\mathbf{p}'\cdot\mathbf{p}}{4M^2}\right) - \frac{i}{4M^2}\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\sigma}\right\}\phi^0_V - \frac{1}{2M}\left\{(\mathbf{p}'+\mathbf{p}) + i(1+\kappa_V)\boldsymbol{\sigma}_1\times\mathbf{k}\right\}\cdot\boldsymbol{\phi}_V\right], \tag{2.3b}$$

$$\bar{u}(\mathbf{p}')\Gamma^{(1)}_S u(\mathbf{p}) = g_S\left[\left(1-\frac{\mathbf{p}'\cdot\mathbf{p}}{4M^2}\right) - \frac{i}{4M^2}\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\sigma}\right], \tag{2.3c}$$

where we defined $\mathbf{k} = \mathbf{p}' - \mathbf{p}$ and $\kappa_V = f_V/g_V$. In the pseudovector vertex, the upper (lower) sign stands for creation (absorption) of the pion at the vertex. In passing we note that the inclusion of the $1/M^2$-terms is necessary in order to get spin-orbit potentials, like in the case of the OBE-potentials.

Assigning always the momentum $\mathbf{k}_1$ to the pion, the NNm_1m_2-vertices (2.2a)-(2.2g) result in

$$\bar{u}(\mathbf{p}')\Gamma^{(2)}_S u(\mathbf{p}) = \frac{g_{(\pi\pi)_0}}{m_\pi}\left[\left(1-\frac{\mathbf{p}'\cdot\mathbf{p}}{4M^2}\right) - \frac{i}{4M^2}\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\sigma}\right], \tag{2.4a}$$

$$\bar{u}(\mathbf{p}')\Gamma^{(2)}_E u(\mathbf{p}) = \frac{g_{(\pi\eta)}}{m_\pi}\left[\left(1-\frac{\mathbf{p}'\cdot\mathbf{p}}{4M^2}\right) - \frac{i}{4M^2}\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\sigma}\right], \tag{2.4b}$$

$$\bar{u}(\mathbf{p}')\Gamma^{(2)}_V u(\mathbf{p}) = i\frac{g_{(\pi\pi)_1}}{m_\pi^2}\left[(\pm\omega_1 \mp \omega_2)\left\{\left(1+\frac{\mathbf{p}'\cdot\mathbf{p}}{4M^2}\right) - \frac{i}{4M^2}\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\sigma}\right\} + \frac{1}{M}\left\{\mathbf{q}\cdot(\mathbf{k}_1-\mathbf{k}_2) - (1+\kappa_1)\boldsymbol{\sigma}\cdot(\mathbf{k}_1\times\mathbf{k}_2)\right\}\right], \tag{2.4c}$$

$$\bar{u}(\mathbf{p}')\Gamma^{(2)}_A u(\mathbf{p}) = g_{(\pi\rho)_1}\left[\boldsymbol{\sigma}\cdot\boldsymbol{\rho} - \frac{1}{M}\boldsymbol{\sigma}\cdot\mathbf{q}\,\rho^0\right]/m_\pi, \tag{2.4d}$$

$$\bar{u}(\mathbf{p}')\Gamma^{(2)}_P u(\mathbf{p}) = i\frac{g_{(\pi\sigma)}}{m_\pi^2}\left[\boldsymbol{\sigma}\cdot(\mathbf{k}_1-\mathbf{k}_2) - \frac{1}{M}\boldsymbol{\sigma}\cdot\mathbf{q}(\pm\omega_1\mp\omega_2)\right], \tag{2.4e}$$

$$\bar{u}(\mathbf{p}')\Gamma^{(2)}_H u(\mathbf{p}) = ig_{(\pi\rho)_0}\left[(\pm\omega_1\pm\omega_2)\boldsymbol{\sigma}\cdot\boldsymbol{\rho} + \boldsymbol{\sigma}\cdot(\mathbf{k}_1+\mathbf{k}_2)\rho^0\right]/m_\pi^2, \tag{2.4f}$$

$$\bar{u}(\mathbf{p}')\Gamma^{(2)}_B u(\mathbf{p}) = ig_{(\pi\omega)}\left[(\pm\omega_1\pm\omega_2)\boldsymbol{\sigma}\cdot\boldsymbol{\omega} + \boldsymbol{\sigma}\cdot(\mathbf{k}_1+\mathbf{k}_2)\omega^0\right]/m_\pi^2, \tag{2.4g}$$

where $\mathbf{q} = \frac{1}{2}(\mathbf{p}'+\mathbf{p})$ and $\kappa_1 = (f/g)_{(\pi\pi)_1}$. Again, the upper (lower) sign in front of ω_1 and ω_2 refers to creation (absorption) of the meson at the vertex. The (πP)-pair can be obtained from the $(\pi\sigma)$-pair simply by making the substitution $g_{(\pi\sigma)} \longrightarrow g_{(\pi P)}$.

The $(\sigma\sigma)$-pair coupling is similar to (2.4a) but with the coupling $g_{(\sigma\sigma)}/m_\pi$, and the $(\pi\eta)$-pair coupling is similar to (2.4b), but with $g_{(\pi\eta')}/m_\pi$.

B. Non-vanishing Effective Meson-Pair YNN-graphs

In Fig. 2 and Fig. 3 are shown the graphs that picture the possible contributions to the *effective hyperon-nucleon potentials* under the condition that the "spectator nucleon" (N_3) does not have a spin-flip nor a flavor change. This means that graphs with a pseudo-scalar meson coupling to this N_3 nucleon can not contribute.

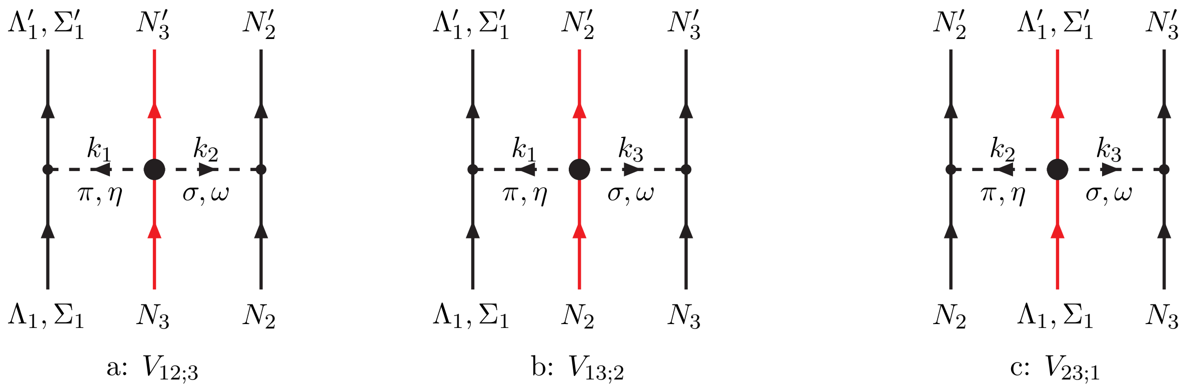

FIG. 2: *The Born-Feynman direct-diagrams for* $V_{12;3}, V_{13;2}, V_{23;1}$

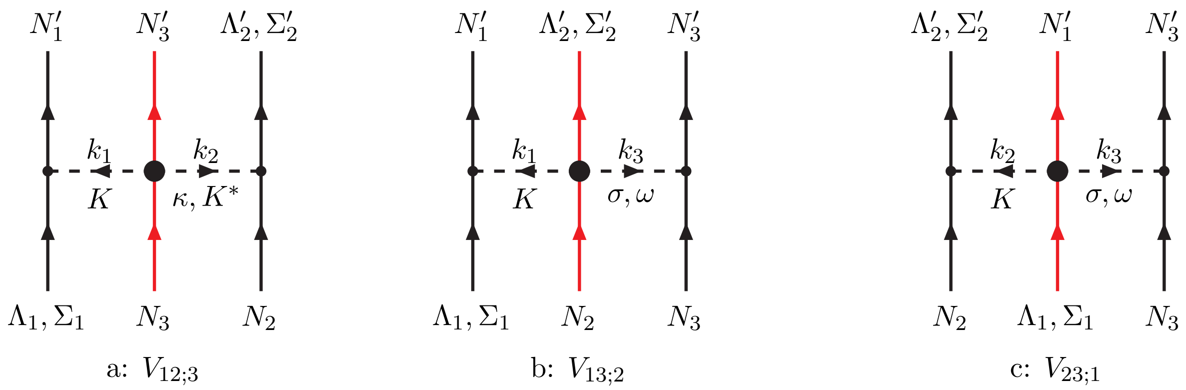

FIG. 3: *The Born-Feynman exchange-diagrams for* $V_{12;3}, V_{13;2}, V_{23;1}$

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III. THE FEYNMAN-DIAGRAM COMPUTATION

In this section we evaluate the Feynman-diagrams in Fig. 2 for two cases for the purpose of illustration.

1. $(\pi\rho)_1$-pair: Application of the Feynman-rules [20] we have for the diagram (a) in Fig. 2

$$\begin{aligned}
-i(2\pi)^4\delta^4(.....)\ V_{12;3} \ =\ & (-i)^3\ g_V \frac{f_P}{m_\pi}\frac{g_{(\pi\rho)_1}}{m_\pi}(\tau_1)_i(\tau_2)_j\epsilon_{ijk}(\tau_3)_k \cdot \\
& \times \int \frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}(+i)\bar{u}(p_1')\gamma_5\left(\gamma\cdot p_1' - \gamma\cdot p_1\right)u(p_1)\cdot \\
& \times \bar{u}(p_3')\gamma_5\gamma_\kappa u(p_3)\bar{u}(p_2')\left[\gamma_\mu + \frac{i}{2\mathcal{M}}\frac{f_V}{g_V}\sigma_{\mu\nu}(p_2'-p_2)^\nu\right]\ u(p_2)\cdot \\
& \times \frac{i}{k_1^2-m_\pi^2+i\epsilon}\frac{-i\eta^{\kappa\mu}}{k_2^2-m_\rho^2+i\epsilon}\cdot(2\pi)^4\delta^4(p_3'-p_3+k_1+k_2)\cdot \\
& \times (2\pi)^4\delta^4(p_1'-p_1-k_1)\ (2\pi)^4\delta^4(p_2'-p_2-k_2)\ , \qquad (3.1)
\end{aligned}$$

where we used

$$\sum_\lambda \epsilon_\rho^\alpha(\lambda)\epsilon_\rho^\mu(\lambda) = -\eta^{\mu\alpha} + \frac{k^\mu k^\alpha}{m_\rho^2} \to -\eta^{\mu\alpha}. \qquad (3.2)$$

Here we anticipated the fact that the vector-meson couples to a conserved current so that the $k^\mu k^\alpha$-term gives no contribution. From (3.1) we obtain

$$\begin{aligned}
V_{12;3} \ =\ & +g_V\frac{f_P}{m_\pi}\frac{g_{(\pi\rho)_1}}{m_\pi}\left(i\ \boldsymbol{\tau}_1\times\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\right)\cdot \\
& \times 2M\left[\bar{u}(p_1')\gamma_5 u(p_2)\right]\ \left[\bar{u}(p_3')\gamma_5\gamma^\mu u(p_3)\right]\ \cdot \\
& \times \bar{u}(p_2')\left[\gamma_\mu + \frac{i}{2\mathcal{M}}\frac{f_V}{g_V}\sigma_{\mu\nu}(p_2'-p_2)^\nu\right]\ u(p_2)\cdot \\
& \times \frac{1}{k_1^2-m_\pi^2+i\epsilon}\frac{1}{k_2^2-m_\rho^2+i\epsilon}\ , \qquad (3.3)
\end{aligned}$$

where because of the δ-functions in (3.1)

$$p_1' + p_2' + p_3' = p_1 + p_2 + p_3\ , \qquad (3.4a)$$

$$k_1 = p_1' - p_1\quad ,\quad k_2 = p_2' - p_2\ , \qquad (3.4b)$$

$$p_3' - p_3 = k_1 + k_2\ . \qquad (3.4c)$$

The exact transition from Dirac spinors to Pauli spinors is given in Appendix A. From the expressions in A, keeping only terms up to order $1/M$, and setting the scaling mass $\mathcal{M} = M$, we find that the vertex operators in Pauli-spinor space for the NNm vertices are given by

$$
\begin{aligned}
\bar{u}(\mathbf{p}')u(\mathbf{p}) &= \left[\left(1-\frac{\mathbf{p}'\cdot\mathbf{p}}{4M^2}\right) - \frac{i}{4M^2}\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\sigma}\right], && (3.5a)\\
\bar{u}(\mathbf{p}')\gamma_5 u(\mathbf{p}) &= -\frac{1}{2M}[\boldsymbol{\sigma}\cdot(\mathbf{p}'-\mathbf{p})] = -\frac{1}{2M}[\boldsymbol{\sigma}\cdot\mathbf{k}], && (3.5b)\\
\bar{u}(\mathbf{p}')\gamma^0 u(\mathbf{p}) &= \left[\left(1+\frac{\mathbf{p}'\cdot\mathbf{p}}{4M^2}\right) + \frac{i}{4M^2}\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\sigma}\right], && (3.5c)\\
\bar{u}(\mathbf{p}')\boldsymbol{\gamma}\, u(\mathbf{p}) &= \frac{1}{2M}\left[(\mathbf{p}'+\mathbf{p}) + i\boldsymbol{\sigma}\times(\mathbf{p}'-\mathbf{p})\right], && (3.5d)\\
\bar{u}(\mathbf{p}')\gamma_5\gamma^0 u(\mathbf{p}) &= -\frac{1}{2M}[\boldsymbol{\sigma}\cdot(\mathbf{p}'+\mathbf{p})] = -\frac{1}{M}[\boldsymbol{\sigma}\cdot\mathbf{q}], && (3.5e)\\
\bar{u}(\mathbf{p}')\gamma_5\boldsymbol{\gamma}\, u(\mathbf{p}) &= -\left[\boldsymbol{\sigma} + \frac{1}{4M^2}(\boldsymbol{\sigma}\cdot\mathbf{p}')\ \boldsymbol{\sigma}\ (\boldsymbol{\sigma}\cdot\mathbf{p})\right] = -\left[\left(1-\frac{\mathbf{p}'\cdot\mathbf{p}}{4M^2}\right)\boldsymbol{\sigma}\right.\\
&\quad \left. -\frac{i}{4M^2}\mathbf{p}'\times\mathbf{p} + \frac{1}{4M^2}\left(\boldsymbol{\sigma}\cdot\mathbf{p}\ \mathbf{p}' + \boldsymbol{\sigma}\cdot\mathbf{p}'\ \mathbf{p}\right)\right] \approx -\boldsymbol{\sigma},, && (3.5f)
\end{aligned}
$$

where we defined $\mathbf{k} = \mathbf{p}' - \mathbf{p}$, $\mathbf{q} = (\mathbf{p}' + \mathbf{p})/2$, and $\kappa_V = f_V/g_V$. In passing we note that the inclusion of the $1/M^2$-terms is necessary in order to get spin-orbit potentials, like in the case of the OBE-potentials.

For the magnetic-coupling we use the Gordon decomposition

$$
i\ \bar{u}(p')\ \sigma^{\mu\nu}(p'-p)_\nu u(p) = \bar{u}(p')\left\{2M\gamma^\mu - (p'+p)^\mu\right\}\ u(p) \qquad (3.6)
$$

We get

$$
\begin{aligned}
& i\ \bar{u}(p')\ \sigma^{\mu\nu}(p'-p)_\nu u(p) \Longrightarrow \\
& \mu = 0\ :\ \frac{1}{M}\left[\mathbf{p}'\cdot\mathbf{p} - \frac{1}{2}(p'^2+p^2) + i\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\sigma}\right],, && (3.7a)\\
& \mu = i\ :\ -\left[\frac{1}{2}(\mathbf{p}'+\mathbf{p}) - \frac{i}{2}\boldsymbol{\sigma}\times(\mathbf{p}'-\mathbf{p})\right]. && (3.7b)
\end{aligned}
$$

For the complete vector-vertex we obtain

$$\bar{u}(p')\Gamma_V^\mu u(p) \equiv \bar{u}(p')\left[\gamma^\mu + \frac{i}{2M}\kappa_V\sigma^{\mu\nu}(p'-p)_\nu\right]u(p)$$

$$= \bar{u}(p')\left[(1+\kappa_V)\gamma^\mu - \frac{\kappa_V}{2M}(p'+p)_\mu\right]u(p) \Longrightarrow$$

$$\mu = 0 \ : \ \left[1+(1+2\kappa_V)\frac{\mathbf{p}'\cdot\mathbf{p}}{4M^2} - \kappa_V\frac{\mathbf{p}'^2+\mathbf{p}^2}{4M^2} + i(1+2\kappa_V)\frac{\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\sigma}}{4M^2}\right], \quad (3.8a)$$

$$\mu = i \ : \ \frac{1}{M}\left[\frac{1}{2}(\mathbf{p}'+\mathbf{p}) + \frac{i}{2}(1+\kappa_V)\boldsymbol{\sigma}\times(\mathbf{p}'-\mathbf{p})\right]. \quad (3.8b)$$

Using these expressions for the vertices, we find from (3.3

$$V_{12;3} \sim +g_V\frac{f_P}{m_\pi}\frac{g_{(\pi\rho)_1}}{m_\pi M}\,(i\,\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)\cdot$$
$$\times(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)\left\{(\boldsymbol{\sigma}_3\cdot\mathbf{q}_3 - \boldsymbol{\sigma}_3\cdot\mathbf{q}_2) + \frac{i}{2}(1+\kappa_V)\,\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_3\cdot\mathbf{k}_2\right.$$
$$\left. -\frac{1}{4M^2}(\boldsymbol{\sigma}_3\cdot\mathbf{q}_3)\left[(1+2\kappa_V)\,\mathbf{p}_2'\cdot\mathbf{p}_2 - \kappa_V\left(p_2'^2+p_2^2\right)\right]\right\}. \quad (3.9)$$

Next, we add to (3.9) the expression for the Feynman-diagram (a) as in Fig. 3 where the pion and rho are interchanged. Then we obtain the potential

$$V_{12;3} = +g_V\frac{f_P}{m_\pi}\frac{g_{(\pi\rho)_1}}{m_\pi M}\,(i\,\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)\left[\left\{(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)\left(\boldsymbol{\sigma}_3\cdot\mathbf{q}_3 - \boldsymbol{\sigma}_3\cdot\mathbf{q}_2\right)\right.\right.$$
$$\left. +\frac{i}{2}(1+\kappa_V)\cdot(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_3\cdot\mathbf{k}_2\right\}\ G^{(0)}(\pi,\mathbf{k}_1;\rho,\mathbf{k}_2)$$
$$-\left\{(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)\left(\boldsymbol{\sigma}_3\cdot\mathbf{q}_3 - \boldsymbol{\sigma}_3\cdot\mathbf{q}_1\right)\right.$$
$$\left.\left. +\frac{i}{2}(1+\kappa_V)\cdot(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_3\cdot\mathbf{k}_1\right\}\ G^{(0)}(\rho,\mathbf{k}_1;\pi,\mathbf{k}_2)\right], \quad (3.10)$$

where

$$G^{(0)}(\pi,\mathbf{k}_1;\rho,\mathbf{k}_2) = F_\pi(\mathbf{k}_1^2)F_\rho(\mathbf{k}_2^2)\ D^{(0)}_{pair}(\omega_\pi(\mathbf{k}_1)\omega_\rho(\mathbf{k}_2)), \quad (3.11)$$

with $\omega_1 = \sqrt{\mathbf{k}_1^2+m_\pi^2}$ and $\omega_2 = \sqrt{\mathbf{k}_2^2+m_\rho^2}$. Furthermore, $D^{(0)}_{pair}(\omega_1,\omega_2) = 1/(\omega_1^2\omega_2^2)$, and $F(\mathbf{k}^2)$ denotes the form factor. Also, in (3.10) we have omitted the terms proportinal to $1/M^3$.

The expressions corresponding to the diagrams (b) and (c) of Fig. 3 can be readily obtained from (3.10) making the appropriate substitutions.

2. $(\pi\pi)_0$-pair: Application of the Feynman-rules [20] for the interaction of the $(\pi\pi)_0$-pairs there are two different contributions.

a) Non-Derivative coupling: Similarly to the expression (3.3) one now has

$$\begin{aligned} V^{(1)}_{12;3} &= -\frac{g^{(1)}_{(\pi\pi)_0}}{m_\pi^3}\left(\frac{f_P}{m_\pi}\right)^2 (\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left[\bar{u}(p_3')u(p_3)\right]\cdot \\ &\times 4M^2\ \left[\bar{u}(p_1')\gamma_5 u(p_2)\right]\ \left[\bar{u}(p_2')\gamma_5 u(p_2)\right]\cdot \\ &\times \frac{1}{k_1^2-m_\pi^2+i\epsilon}\frac{1}{k_2^2-m_\pi^2+i\epsilon}, \end{aligned} \tag{3.12}$$

which gives, for the leading $(1/M)^0$-terms,

$$V^{(1)}_{12;3} = -\frac{g^{(1)}_{(\pi\pi)_0}}{m_\pi^3}\left(\frac{f_P}{m_\pi}\right)^2 (\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\ \boldsymbol{\sigma}_2\cdot\mathbf{k}_2\right)\ G^{(0)}(\pi,\mathbf{k}_1;\pi,\mathbf{k}_2), \tag{3.13}$$

where

$$G^{(0)}(\pi,\mathbf{k}_1;\pi,\mathbf{k}_2) = F_\pi(\mathbf{k}_1^2)F_\pi(\mathbf{k}_2^2)\ D^{(0)}_{pair}(\omega_\pi(\mathbf{k}_1,\ \omega_\pi(\mathbf{k}_2). \tag{3.14}$$

b) Derivative coupling: Similarly to the expression (3.3) one now has

$$\begin{aligned} V^{(2)}_{12;3} &= +\frac{g^{(2)}_{(\pi\pi)_0}}{m_\pi^3}\left(\frac{f_P}{m_\pi}\right)^2 (\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(k_1\cdot k_2\right)\left[\bar{u}(p_3')u(p_3)\right]\cdot \\ &\times 4M^2\ \left[\bar{u}(p_1')\gamma_5 u(p_2)\right]\ \left[\bar{u}(p_2')\gamma_5 u(p_2)\right]\cdot \\ &\times \frac{1}{k_1^2-m_\pi^2+i\epsilon}\frac{1}{k_2^2-m_\pi^2+i\epsilon}, \end{aligned} \tag{3.15}$$

which gives, for the leading $(1/M)^0$-terms,

$$\begin{aligned} V^{(2)}_{12;3} &= +\frac{g^{(2)}_{(\pi\pi)_0}}{m_\pi^3}\left(\frac{f_P}{m_\pi}\right)^2 (\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1\ \boldsymbol{\sigma}_2\cdot\mathbf{k}_2\right)\cdot \\ &\times\left[\omega_1\omega_2-\mathbf{k}_1\cdot\mathbf{k}_2\right]\ G^{(0)}(\pi,\mathbf{k}_1;\pi,\mathbf{k}_2). \end{aligned} \tag{3.16}$$

Note that in both cases there is an extra factor 2 because of "identical particles" comming from the matrix element of the $\boldsymbol{\pi}\cdot\boldsymbol{\pi}$*-operator.*

The generalization of the interaction kernels to the case with a Gaussian (or any other) form factor has been treated and explained in [19]. We make the substitution

$$[k^2-m^2+i\delta]^{-1} \longrightarrow \int_0^\infty d\mu^2 \frac{\rho(\mu^2)}{k^2-\mu^2+i\delta}, \tag{3.17}$$

for each meson-exchange line in the Feynman diagrams. Here, $\rho(\mu^2)$ is the spectral function, representing the form factors involved in meson exchange. At low and medium energy, we have to a very good approximation $t = k^2 \approx -\mathbf{k}^2 < 0$, and so for space-like momentum transfers we can use Gaussian form factors $F(\mathbf{k}^2) = \exp(-\mathbf{k}^2/\Lambda^2)$, where Λ denotes the cutoff mass. The Gaussian form factor is introduced by the substitution

$$\int_0^\infty d\mu^2 \frac{\rho(\mu^2)}{\mathbf{k}^2+\mu^2} \longrightarrow \frac{F(\mathbf{k}^2)}{\mathbf{k}^2+m^2}. \tag{3.18}$$

The NNm and NNm_1m_2 vertices have different form factors. We will use

$$F_{NNm_1m_2}(\mathbf{k}_1,\mathbf{k}_2) = \exp(-\mathbf{k}_1^2/2\Lambda_1^2)\,\exp(-\mathbf{k}_2^2/2\Lambda_2^2),$$
$$F_{NNm}(\mathbf{k}^2) = \exp(-\mathbf{k}^2/2\Lambda_m^2), \tag{3.19}$$

where Λ_1 and Λ_2 are the form factor masses for mesons m_1 and m_2, respectively. A motivation for this prescription could be that in "duality" the structure of the NNm_1m_2 vertex is either saturated by heavy mesons or meson-nucleon resonances. In this last case, assuming that the meson-nucleon resonance transitions all have roughly the same (inelastic) form factor, the form (3.19) is a natural one.

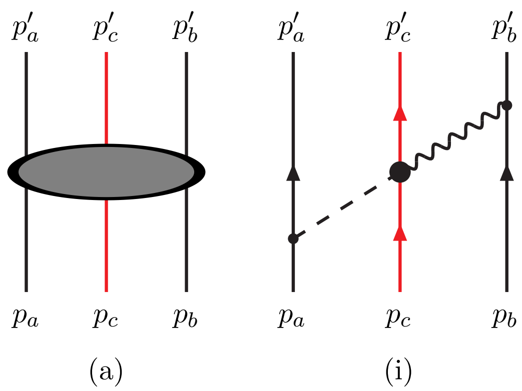


FIG. 4: *Three-particle amplitude (a) and the Born-Feynman graph (i)*

IV. THREE-PARTICLE POTENTIAL IN CONFIGURATION SPACE

The potential in configuration space is given by the Fourier transform

$$V\left(\mathbf{x}'_1,\mathbf{x}'_2,\mathbf{x}'_3;\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3\right) = \Pi_{i=1,3}\left[\int \frac{d^3p'_i}{(2\pi)^3}\right]\Pi_{j=1,3}\left[\int \frac{d^3p_j}{(2\pi)^3}\right]\cdot$$
$$\times\exp\left(-i\sum_{i=1,3}\mathbf{p}'_i\cdot\mathbf{x}'_i\right)\exp\left(+i\sum_{i=1,3}\mathbf{p}_i\cdot\mathbf{x}_i\right)\cdot\widetilde{V}\left(\mathbf{p}'_1,\mathbf{p}'_2,\mathbf{p}'_3;\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3\right) \tag{4.1}$$

Introducing the standard set of variables

$$\mathbf{q}_i = \frac{1}{2}(\mathbf{p}_i + \mathbf{p}'_i) \ , \ \mathbf{k}_i = \mathbf{p}'_i - \mathbf{p}_i \ \ , \ \ \sum_{i=1,3} \mathbf{k}_i = 0 \ , \tag{4.2}$$

where the last condition comes from translation invariance momentum conservation. In terms of these momenta one has

$$\mathbf{p}_i \cdot \mathbf{x}_i - \mathbf{p}'_i \cdot \mathbf{x}'_i \ = \ \mathbf{q}_i \cdot (\mathbf{x}_i - \mathbf{x}'_i) - \frac{1}{2}\mathbf{k}_i \cdot (\mathbf{x}_i + \mathbf{x}'{}_i) \ , \tag{4.3}$$

and therefore if the potential depends only the $\mathbf{k}_i$-variables, i.e.

$$\widetilde{V} = \widetilde{V}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \tag{4.4}$$

the d^3q_i-integrals give $\delta(\mathbf{r}_i - \mathbf{r}'_i)$ and one obtains

$$\begin{aligned} V\left(\mathbf{x}'_1, \mathbf{x}'_2, \mathbf{x}'_3; \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\right) \ &= \ \delta(\mathbf{x}'_1 - \mathbf{x}_1)\delta(\mathbf{x}'_2 - \mathbf{x}_2)\delta(\mathbf{x}'_3 - \mathbf{x}_3) \cdot \\ &\times \Pi_{i=1,3} \left[\int \frac{d^3k_i}{(2\pi)^3} e^{-i\mathbf{k}_i \cdot \mathbf{x}_i} \right] \ . \end{aligned} \tag{4.5}$$

In the following sections we calculate the MPE-potentials in configuration space. In momentum space the MPE-potentials will be of the general form

$$\widetilde{V}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \ = \ (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \ \widetilde{V}(\mathbf{k}_1, \mathbf{k}_2) \tag{4.6}$$

Application of the simple result (4.5) to the potentials of the form (4.6) gives

$$\begin{aligned} & V\left(\mathbf{x}'_1, \mathbf{x}'_2, \mathbf{x}'_3; \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\right) = \delta(\mathbf{x}'_1 - \mathbf{x}_1)\delta(\mathbf{x}'_2 - \mathbf{x}_2)\delta(\mathbf{x}'_3 - \mathbf{x}_3) \cdot \\ & \times \int \frac{d^3k_1}{(2\pi)^3} \int \frac{d^3k_2}{(2\pi)^3} e^{-i\mathbf{k}_1 \cdot (\mathbf{x}_1 - \mathbf{x}_3)} e^{-i\mathbf{k}_2 \cdot (\mathbf{x}_2 - \mathbf{x}_3)} \widetilde{V}(\mathbf{k}_1, \mathbf{k}_2) \ . \end{aligned} \tag{4.7}$$

In the following we will denote the position of particle 3 also by $\mathbf{r}_3 = \mathbf{x}_3$, and denote the relative distances between particles 1 and 2 with respect to particle 3 by $\mathbf{r}_1$ and $\mathbf{r}_2$. So

$$\mathbf{r}_1 = \mathbf{x}_1 - \mathbf{x}_3 \ \ , \ \ \mathbf{r}_2 = \mathbf{x}_2 - \mathbf{x}_3 \ \ , \ \ \mathbf{r}_3 = \mathbf{x}_3 \ . \tag{4.8}$$

Here, a note on the partial derivatives is in order. Since we have by (4.6), and consequently (4.7), that the functions we deal with are of the type $v(\mathbf{x}_1 - \mathbf{x}_3, \mathbf{x}_2 - \mathbf{x}_3) \equiv v(\mathbf{r}_1, \mathbf{r}_2)$ and therefore

$$\nabla_{1,i} \equiv \nabla_{\mathbf{x}_1,i} \ = \ \nabla_{\mathbf{r}_1,i} \ \ , \ \ \nabla_{2,i} \equiv \nabla_{\mathbf{x}_2,i} = \nabla_{\mathbf{r}_2,i} \ , \tag{4.9a}$$

$$\nabla_{3,i} \equiv \nabla_{\mathbf{x}_3,i} \ = \ -\left(\nabla_{\mathbf{r}_1,i} + \nabla_{\mathbf{r}_2,i}\right) \ . \tag{4.9b}$$

V. TWO-MESON-PAIR EXCHANGE POTENTIAL IN MOMENTUM-SPACE

To cover all three graphs in Fig. 3, we have starting from the results of section III for the $\widetilde{V}_{12;3}$-potentials to make the appropriate substitutions. This implies that w.r.t. the first term, for the second and third term we have to change labels of the isospin, spin, and external momenta p_i. The exchange momenta k_j are unchanged. So, we have the scheme given in Table I.

	$1 \to 1$	$\tau_1 \to \tau_1$	$\sigma_1 \to \sigma_1$	$\mathbf{k}_1 \to \mathbf{k}_1$
$V_{12;3} \to V_{13;2}$:	$2 \to 3$	$\tau_2 \to \tau_3$	$\sigma_2 \to \sigma_3$	$\mathbf{k}_2 \to \mathbf{k}_3$
	$3 \to 2$	$\tau_3 \to \tau_2$	$\sigma_3 \to \sigma_2$	$\mathbf{k}_3 \to \mathbf{k}_2$
	$1 \to 2$	$\tau_1 \to \tau_2$	$\sigma_1 \to \sigma_2$	$\mathbf{k}_1 \to \mathbf{k}_2$
$V_{12;3} \to V_{23;1}$:	$2 \to 3$	$\tau_2 \to \tau_3$	$\sigma_2 \to \sigma_3$	$\mathbf{k}_2 \to \mathbf{k}_3$
	$3 \to 1$	$\tau_3 \to \tau_1$	$\sigma_3 \to \sigma_1$	$\mathbf{k}_3 \to \mathbf{k}_1$

TABLE I: Substitution rules for permutation of the external nucleons.

(i) $J^{PC} = 0^{++}$ $(\pi\pi)_0$-Pair Exchange Potential: The adiabatic potentials $\widetilde{V}_{12:3}$ for this pair-interaction are

$$\left(\widetilde{V}^{(1)}_{12;3}\right)^{(0)}(\mathbf{k}_1,\mathbf{k}_2) = -\frac{g^{(1)}_{(\pi\pi)_0}}{m_\pi}\left(\frac{f}{m_\pi}\right)^2 (\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\ (\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)\cdot \times F(\mathbf{k}_1^2)F(\mathbf{k}_2^2)\ D^{(0)}_{pair}(\omega_1,\omega_2), \tag{5.1a}$$

$$\left(\widetilde{V}^{(2)}_{12;3}\right)^{(0)}(\mathbf{k}_1,\mathbf{k}_2) = +\frac{g^{(2)}_{(\pi\pi)_0}}{m_\pi^3}\left(\frac{f}{m_\pi}\right)^2 (\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\ (\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)\cdot \times\left[\omega_1\omega_2-\mathbf{k}_1\cdot\mathbf{k}_2\right]F(\mathbf{k}_1^2)F(\mathbf{k}_2^2)\ D^{(0)}_{pair}(\omega_1,\omega_2), \tag{5.1b}$$

where

$$D^{(0)}_{pair}(\omega_1,\omega_2) \equiv D^{(1),0}_{pair}(\omega_1,\omega_2) = D^{(2),0}_{pair}(\omega_1,\omega_2) = \frac{1}{\omega_1^2\omega_2^2} \tag{5.2}$$

(ii) $J^{PC}=1^{--}$ $(\pi\pi)_1$-Pair Exchange Potential: ‘

$$\left(\widetilde{V}_{12;3}\right)^{(1)}_{(\pi\pi)_1} = +i\frac{g_{(\pi\pi)_1}}{m_\pi^4}\left(\frac{f_P^2}{2M}\right)(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)\cdot \times\left[\mathbf{q}_1\cdot\mathbf{k}_1-\mathbf{q}_2\cdot\mathbf{k}_2\right]\ F(\mathbf{k}_1^2)F(\mathbf{k}_2^2)\ D^{(0)}_{pair}(\omega_1,\omega_2)\ , \tag{5.3a}$$

$$\left(\widetilde{V}_{12;3}\right)^{(2)}_{(\pi\pi)_1} = -i\frac{g_{(\pi\pi)_1}}{m_\pi^4}\left(\frac{f_P^2}{2M}\right)(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)\cdot \times\left[\frac{1}{2}(\mathbf{p}_3'+\mathbf{p}_3)\cdot(\mathbf{k}_1-\mathbf{k}_2)+(1+\kappa_1)\ \boldsymbol{\sigma}_3\cdot(\mathbf{k}_1\times\mathbf{k}_2)\right]\cdot \times F(\mathbf{k}_1^2)F(\mathbf{k}_2^2)\ D^{(0)}_{pair}(\omega_1,\omega_2)\ . \tag{5.3b}$$

(iii) $J^{PC}=1^{++}$ $(\pi\rho)_1$-Pair Exchange Potential:

$$\left(\widetilde{V}_{12;3}\right)^{(1)}_{(\pi\rho)_1} = +i\frac{g_{(\pi\rho)_1}}{m_\pi}\left(\frac{f_P g_V}{2m_\pi M}\right)(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)\left[\left\{(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)\ \boldsymbol{\sigma}_3\cdot\left((\mathbf{p}_3'-\mathbf{p}_2')+(\mathbf{p}_3-\mathbf{p}_2)\right) - i(1+\kappa_V)(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)(\boldsymbol{\sigma}_3\cdot\boldsymbol{\sigma}_2\times\mathbf{k}_2)\right\}\ G(\pi,\mathbf{k}_1;\rho,\mathbf{k}_2) - \left\{(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)\ \boldsymbol{\sigma}_3\cdot\left((\mathbf{p}_3'-\mathbf{p}_1')+(\mathbf{p}_3-\mathbf{p}_1)\right) - i(1+\kappa_V)(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)(\boldsymbol{\sigma}_3\cdot\boldsymbol{\sigma}_1\times\mathbf{k}_1)\right\}\ G(\rho,\mathbf{k}_1;\pi,\mathbf{k}_2)\right]. \tag{5.4}$$

$$\left(\widetilde{V}_{13;2}\right)^{(1)}_{(\pi\rho)_1} = +i\frac{g_{(\pi\rho)_1}}{m_\pi}\left(\frac{f_P g_V}{2m_\pi M^2}\right)(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_3\cdot\boldsymbol{\tau}_2)\left[\left\{(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)\ \boldsymbol{\sigma}_2\cdot\left((\mathbf{p}_2'-\mathbf{p}_3')+(\mathbf{p}_2-\mathbf{p}_3)\right) - i(1+\kappa_V)(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)(\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3\times\mathbf{k}_3)\right\}\ G(\pi,\mathbf{k}_1;\rho,\mathbf{k}_3) - \left\{(\boldsymbol{\sigma}_3\cdot\mathbf{k}_3)\ \boldsymbol{\sigma}_3\cdot\left((\mathbf{p}_2'-\mathbf{p}_1')+(\mathbf{p}_2-\mathbf{p}_1)\right) -i(1+\kappa_V)(\boldsymbol{\sigma}_3\cdot\mathbf{k}_3)(\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_1\times\mathbf{k}_1)\right\}\ G(\rho,\mathbf{k}_1;\pi,\mathbf{k}_3)\right]. \tag{5.5}$$

$$\left(\widetilde{V}_{23;1}\right)^{(1)}_{(\pi\rho)_1} = +i\frac{g_{(\pi\rho)_1}}{m_\pi}\left(\frac{f_P g_V}{2m_\pi M^2}\right)(\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3\cdot\boldsymbol{\tau}_1)\left[\left\{(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)\ \boldsymbol{\sigma}_1\cdot\left((\mathbf{p}_1'-\mathbf{p}_3')+(\mathbf{p}_1-\mathbf{p}_3)\right) - i(1+\kappa_V)(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_3\times\mathbf{k}_3)\right\}\ G(\pi,\mathbf{k}_2;\rho,\mathbf{k}_3) - \left\{(\boldsymbol{\sigma}_3\cdot\mathbf{k}_3)\ \boldsymbol{\sigma}_1\cdot\left((\mathbf{p}_1'-\mathbf{p}_2')+(\mathbf{p}_1-\mathbf{p}_2)\right) -i(1+\kappa_V)(\boldsymbol{\sigma}_3\cdot\mathbf{k}_3)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\times\mathbf{k}_2)\right\}\ G(\rho,\mathbf{k}_2;\pi,\mathbf{k}_3)\right]. \tag{5.6}$$

We note that these expresions can be used for the pair $(\pi\rho)_0$ by replacing the isospin factor in front by $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$, $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3$, and $\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3$, for respectively (5.4), (5.5), and (5.6).

(iv) $J^{PC}=1^{++}$ $(\pi\sigma)_1$-Pair Exchange Potential:

$$\begin{aligned}\left(\widetilde{V}_{12;3}\right)^{(1)}_{(\pi\sigma)_1} &= +\frac{g_{(\pi\sigma)_1}}{m_\pi^2}\left(\frac{f_P g_S}{m_\pi}\right)\Big[(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)(\boldsymbol{\sigma}_3\cdot(\mathbf{k}_1-\mathbf{k}_2))(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3)\ G(\pi,\mathbf{k}_1;\sigma,\mathbf{k}_2)\\ &\quad +(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)(\boldsymbol{\sigma}_3\cdot(\mathbf{k}_2-\mathbf{k}_1)(\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)\ G(\sigma,\mathbf{k}_1;\pi,\mathbf{k}_2)\Big]\,. \qquad (5.7a)\\ \left(\widetilde{V}_{12;3}\right)^{(2)}_{(\pi\sigma)_1} &= -\frac{g_{(\pi\sigma)_1}}{m_\pi^2}\left(\frac{f_P g_S}{2m_\pi M^2}\right)\Big[(\boldsymbol{\sigma}_3\cdot(\mathbf{p}_3'+\mathbf{p}_3))(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3)\ G(\pi,\mathbf{k}_1;\sigma,\mathbf{k}_2)\\ &\quad +(\boldsymbol{\sigma}_3\cdot(\mathbf{p}_3'+\mathbf{p}_3))(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)(\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)\ G(\sigma,\mathbf{k}_1;\pi,\mathbf{k}_2)\Big]\,\cdot\\ &\quad \times\Big[2\mathbf{q}_1\cdot\mathbf{k}_1+\mathbf{k}_1^2-2\mathbf{q}_2\cdot\mathbf{k}_2+\mathbf{k}_2^2\Big] \qquad (5.7b)\end{aligned}$$

$$\begin{aligned}\left(\widetilde{V}_{13;2}\right)^{(1)}_{(\pi\sigma)_1} &= +\frac{g_{(\pi\sigma)_1}}{m_\pi^2}\left(\frac{f_P g_S}{m_\pi}\right)\Big[(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)(\boldsymbol{\sigma}_2\cdot(\mathbf{k}_1-\mathbf{k}_3))(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\ G(\pi,\mathbf{k}_1;\sigma,\mathbf{k}_3)\\ &\quad +(\boldsymbol{\sigma}_3\cdot\mathbf{k}_3)(\boldsymbol{\sigma}_2\cdot(\mathbf{k}_3-\mathbf{k}_1))(\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)G(\sigma,\mathbf{k}_1;\pi,\mathbf{k}_3)\Big]\,, \qquad (5.8a)\\ \left(\widetilde{V}_{13;2}\right)^{(2)}_{(\pi\sigma)_1} &= -\frac{g_{(\pi\sigma)_1}}{m_\pi^2}\left(\frac{f_P g_S}{2m_\pi M^2}\right)\Big[(\boldsymbol{\sigma}_2\cdot(\mathbf{p}_2'+\mathbf{p}_2))(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\ G(\pi,\mathbf{k}_1;\sigma,\mathbf{k}_3)\\ &\quad +(\boldsymbol{\sigma}_2\cdot(\mathbf{p}_2'+\mathbf{p}_2))(\boldsymbol{\sigma}_3\cdot\mathbf{k}_3)(\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)\ G(\sigma,\mathbf{k}_1;\pi,\mathbf{k}_3)\Big]\,\cdot\\ &\quad \times\Big[2\mathbf{q}_1\cdot\mathbf{k}_1+\mathbf{k}_1^2-2\mathbf{q}_3\cdot\mathbf{k}_3+\mathbf{k}_3^2\Big] \qquad (5.8b)\end{aligned}$$

$$\begin{aligned}\left(\widetilde{V}_{23;1}\right)^{(1)}_{(\pi\sigma)_1} &= +\frac{g_{(\pi\sigma)_1}}{m_\pi^2}\left(\frac{f_P g_S}{m_\pi}\right)\Big[(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)(\boldsymbol{\sigma}_1\cdot(\mathbf{k}_2-\mathbf{k}_3))(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\ G(\pi,\mathbf{k}_2;\sigma,\mathbf{k}_3)\\ &\quad +(\boldsymbol{\sigma}_3\cdot\mathbf{k}_3)(\boldsymbol{\sigma}_1\cdot(\mathbf{k}_3-\mathbf{k}_2))(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3)\ G(\sigma,\mathbf{k}_2;\pi,\mathbf{k}_3)\Big]\,, \qquad (5.9a)\\ \left(\widetilde{V}_{23;1}\right)^{(2)}_{(\pi\sigma)_1} &= -\frac{g_{(\pi\sigma)_1}}{m_\pi^2}\left(\frac{f_P g_S}{2m_\pi M^2}\right)\Big[(\boldsymbol{\sigma}_1\cdot(\mathbf{p}_1'+\mathbf{p}_1))(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\ G(\pi,\mathbf{k}_2;\sigma,\mathbf{k}_3)\\ &\quad +(\boldsymbol{\sigma}_1\cdot(\mathbf{p}_1'+\mathbf{p}_1))(\boldsymbol{\sigma}_3\cdot\mathbf{k}_3)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3)\ G(\sigma,\mathbf{k}_2;\pi,\mathbf{k}_3)\Big]\ \cdot\\ &\quad \times\Big[2\mathbf{q}_2\cdot\mathbf{k}_2+\mathbf{k}_2^2-2\mathbf{q}_3\cdot\mathbf{k}_3+\mathbf{k}_3^2\Big] \qquad (5.9b)\end{aligned}$$

(v) $J^{PC} = 1^{+-}$ $(\pi\omega)_1$-Pair Exchange Potential:

$$\left(\widetilde{V}_{12;3}\right)^{(1)}_{(\pi\omega)_1} = -\frac{g_{(\pi\omega)_1}}{m_\pi^2}\left(\frac{f_P g_V}{2m_\pi M^2}\right)\cdot$$
$$\times\left[\boldsymbol{\sigma}_3\cdot\left((\mathbf{p}'_1+\mathbf{p}_1)+i(1+\kappa_V)\ \boldsymbol{\sigma}_1\times\mathbf{k}_1\right)(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)(\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)\ G(\omega,\mathbf{k}_1;\pi,\mathbf{k}_2)\right.$$
$$\left.+\boldsymbol{\sigma}_3\cdot\left((\mathbf{p}'_2+\mathbf{p}_2)+i(1+\kappa_V)\ \boldsymbol{\sigma}_2\times\mathbf{k}_2\right)(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3)\ G(\pi,\mathbf{k}_1;\omega,\mathbf{k}_2)\right]\cdot$$
$$\times\left[\left(2\mathbf{p}_1\cdot\mathbf{k}_1+\mathbf{k}_1^2\right)+\left(2\mathbf{p}_2\cdot\mathbf{k}_2+\mathbf{k}_2^2\right)\right]. \tag{5.10a}$$

(5.10b)

$$\left(\widetilde{V}_{12;3}\right)^{(2)}_{(\pi\omega)_1} = -\frac{g_{(\pi\omega)_1}}{m_\pi^2}\left(\frac{f_P g_V}{m_\pi}\right)\ \boldsymbol{\sigma}_3\cdot(\mathbf{k}_1+\mathbf{k}_2)\cdot$$
$$\times\left[(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3)\ G(\omega,\mathbf{k}_1;\pi,\mathbf{k}_2)+(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)(\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)\ G(\pi,\mathbf{k}_1;\omega,\mathbf{k}_2)\right]. \tag{5.10c}$$

$$\left(\widetilde{V}_{13;2}\right)^{(1)}_{(\pi\omega)_1} = -\frac{g_{(\pi\omega)_1}}{m_\pi^2}\left(\frac{f_P g_V}{2m_\pi M^2}\right)\cdot$$
$$\times\left[\boldsymbol{\sigma}_2\cdot\left((\mathbf{p}'_1+\mathbf{p}_1)+i(1+\kappa_V)\ \boldsymbol{\sigma}_1\times\mathbf{k}_1\right)(\boldsymbol{\sigma}_3\cdot\mathbf{k}_3)(\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)\ G(\omega,\mathbf{k}_1;\pi,\mathbf{k}_3)\right.$$
$$\left.+\boldsymbol{\sigma}_2\cdot\left((\mathbf{p}'_3+\mathbf{p}_3)+i(1+\kappa_V)\ \boldsymbol{\sigma}_3\times\mathbf{k}_3\right)(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\ G(\pi,\mathbf{k}_1;\omega,\mathbf{k}_3)\right]\cdot$$
$$\times\left[\left(2\mathbf{p}_1\cdot\mathbf{k}_1+\mathbf{k}_1^2\right)+\left(2\mathbf{p}_3\cdot\mathbf{k}_3+\mathbf{k}_3^2\right)\right] \tag{5.11a}$$

(5.11b)

$$\left(\widetilde{V}_{13;2}\right)^{(2)}_{(\pi\omega)_1} = -\frac{g_{(\pi\omega)_1}}{m_\pi^2}\left(\frac{f_P g_V}{m_\pi}\right)\ \boldsymbol{\sigma}_2\cdot(\mathbf{k}_1+\mathbf{k}_3)\cdot$$
$$\times\left[(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\ G(\omega,\mathbf{k}_1;\pi,\mathbf{k}_2)+(\boldsymbol{\sigma}_3\cdot\mathbf{k}_3)(\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)\ G(\pi,\mathbf{k}_1;\omega,\mathbf{k}_3)\right] \tag{5.11c}$$

$$\left(\widetilde{V}_{23;1}\right)^{(1)}_{(\pi\omega)_1} = -\frac{g_{(\pi\omega)_1}}{m_\pi^2}\left(\frac{f_P g_V}{2m_\pi M^2}\right)\cdot$$
$$\times\left[\boldsymbol{\sigma}_1\cdot\left((\mathbf{p}_2'+\mathbf{p}_2)+i(1+\kappa_V)\ \boldsymbol{\sigma}_2\times\mathbf{k}_2\right)(\boldsymbol{\sigma}_3\cdot\mathbf{k}_3)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3)\ G(\omega,\mathbf{k}_2;\pi,\mathbf{k}_3)\right.$$
$$\left.+\boldsymbol{\sigma}_1\cdot\left((\mathbf{p}_3'+\mathbf{p}_3)+i(1+\kappa_V)\ \boldsymbol{\sigma}_3\times\mathbf{k}_3\right)(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\ G(\pi,\mathbf{k}_2;\omega,\mathbf{k}_3)\right]\cdot$$
$$\times\left[\left(2\mathbf{p}_2\cdot\mathbf{k}_2+\mathbf{k}_2^2\right)+\left(2\mathbf{p}_3\cdot\mathbf{k}_3+\mathbf{k}_3^2\right)\right] \tag{5.12a}$$

(5.12b)

$$\left(\widetilde{V}_{23;1}\right)^{(2)}_{(\pi\omega)_1} = -\frac{g_{(\pi\omega)_1}}{m_\pi^2}\left(\frac{f_P g_V}{m_\pi}\right)\boldsymbol{\sigma}_1\cdot(\mathbf{k}_2+\mathbf{k}_3)\cdot$$
$$\times\left[(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\ G(\omega,\mathbf{k}_2;\pi,\mathbf{k}_3)+(\boldsymbol{\sigma}_3\cdot\mathbf{k}_3)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3)\ G(\pi,\mathbf{k}_1;\omega(\mathbf{k}_3)\right] \tag{5.12c}$$

(vi) $J^{PC}=0^{++}$ $(\sigma\sigma)$-Pair Exchange Potential:

$$\widetilde{V}^0_{12;3}(\mathbf{k}_1,\mathbf{k}_2) = -\frac{g_{(\sigma\sigma)}}{m_\pi}\ g_S^2 F(\mathbf{k}_1^2)F(\mathbf{k}_2^2)\ D^{(0)}_{pair}(\omega_1,\omega_2)\ . \tag{5.13}$$

VI. EFFECTIVE TWO-NUCLEON POTENTIALS

The complete three-nucleon MPE-potential is obtained by a sum over the permutations. First summing over the cycles (123 gives

$$V_3(MPE) = V_{12;3}(MPE) + V_{31;2}(MPE) + V_{23;1}(MPE)\ . \tag{6.1}$$

A. Momentum conservation restrictions.

1. In the Three-particle Feynman amplitude there is 3-momentum conservation, i.e.

$$\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3 = \mathbf{p}_1'+\mathbf{p}_2'+\mathbf{p}_3'. \tag{6.2}$$

In the Three-particle CM-system, one has moreover:

$$\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3 = \mathbf{p}_1'+\mathbf{p}_2'+\mathbf{p}_3' = 0, \tag{6.3}$$

and then $\mathbf{q}_3 = -(\mathbf{q}_1 + \mathbf{q}_2)$.

2. In the effective two-particle potential we **must require momentum conservation**, i.e.

$$\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}'_1 + \mathbf{p}'_2 = \mathbf{P}'. \tag{6.4}$$

Combined with (6.2) this implies that $\mathbf{p}_3 = \mathbf{p}'_3$, or $\mathbf{k}_3 = 0$. From (6.4) it follows that

$$\mathbf{q}_1 + \mathbf{q}_2 = \mathbf{P}. \tag{6.5}$$

3. For the effective two-nucleon potential we are going to work in the two-particle CM-system, so

$$\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2) = \mathbf{p}'_1 + \mathbf{p}'_2 = 0, \quad \mathbf{p}'_3 = \mathbf{p}_3. \tag{6.6}$$

The last relation follows from (6.2). This implies $\mathbf{q}_2 = -\mathbf{q}_1$, and $\mathbf{k}_2 = -\mathbf{k}_1, \mathbf{k}_3 = 0$. Then,

$$\widetilde{V}(\mathbf{p}'_1, \mathbf{p}'_2, \mathbf{p}'_3; \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \equiv \widetilde{V}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3; \mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3) \Rightarrow \widetilde{V}(\mathbf{k}_1, -\mathbf{k}_1, \mathbf{0}; \mathbf{q}_1, -\mathbf{q}_1, \mathbf{q}_3). \tag{6.7}$$

In nuclear matter one averages over $\mathbf{p}_3 = \mathbf{q}_3$ [22]. This means that only the quadratic term survives and is replaced by

$$p_3^2 = q_3^2 \rightarrow \langle p_3^2 \rangle = \frac{3}{5} k_F^2, \quad p_{3,i} p_{3,j} \rightarrow \frac{1}{3} \langle p_3^2 \rangle \delta_{ij} = \frac{1}{5} k_F^2. \tag{6.8}$$

B. Effective Two-Nucleon Configuration-space Potentials

A very important application is the structure of the *effective two-nucleon potential*, which can be used in computations of nuclear matter and for finite nuclei in e.g. local-density-approximation (LDA). This is obtained by integrating out, for example, nucleon 3. Using (4.7) we have

$$V_{12;3}^{(eff)} = \frac{1}{4} \rho_{NM}\ Tr \int d^3x_3\ V(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) = \frac{1}{4} \rho_{NM}\ Tr \int d^3x_3 \int \frac{d^3k_1}{(2\pi)^3} \int \frac{d^3k_2}{(2\pi)^3} \cdot$$
$$\times e^{-i\mathbf{k}_1 \cdot (\mathbf{x}_1 - \mathbf{x}_3)} e^{-i\mathbf{k}_2 \cdot (\mathbf{x}_2 - \mathbf{x}_3)} \widetilde{V}(\mathbf{k}_1, \mathbf{k}_2) \left|_{\mathbf{k}_3 = -\mathbf{k}_1 - \mathbf{k}_2}\right. = \frac{1}{4} \rho_{NM}\ Tr \int \frac{d^3k}{(2\pi)^3} e^{-i\mathbf{k} \cdot (\mathbf{x}_1 - \mathbf{x}_2)} \widetilde{V}(\mathbf{k}, -\mathbf{k}), \tag{6.9}$$

where $\rho_{NM} = 4\rho_0 = 4k_F^3/6\pi^2$, with $k_F = 1.4\ \text{fm}^{-1}$ for symmetric nuclear matter.

In (6.9) the Tr-symbol stands for the trace over the spin and isospin operators of particle 3 (LNR-approximation [21, 22]). This amounts to the approximation neglecting *spin-flip and charge change for particle 3. However, this still leaves the operators* $\sigma_{3,z}, \tau_{3,z}$. *The requirement of rotational- and isospin- invariance for the effective 2-body potential, eliminates these operators. This fully justifies the Tr-operation. Note that this is a rather drastic approximation eliminating many terms, because:* $\boldsymbol{\sigma}_3, \boldsymbol{\tau}_3 \to 0$. *For the reduced potential forms: see Appendix VII.*

$V_{12;3}$	$V_{13;2}$	$V_{23;1}$
$\exp[-i\mathbf{k}_1 \cdot (\mathbf{x}_1 - \mathbf{x}_3)]$ $\times \exp[-i\mathbf{k}_2 \cdot (\mathbf{x}_2 - \mathbf{x}_3)]$	$\exp[-i\mathbf{k}_1 \cdot (\mathbf{x}_1 - \mathbf{x}_2)]$ $\times \exp[-i\mathbf{k}_3 \cdot (\mathbf{x}_3 - \mathbf{x}_2)]$	$\exp[-i\mathbf{k}_2 \cdot (\mathbf{x}_2 - \mathbf{x}_1)]$ $\times \exp[-i\mathbf{k}_3 \cdot (\mathbf{x}_3 - \mathbf{x}_1)]$
$\delta(\mathbf{k}_1 + \mathbf{k}_2)$ $\times \exp[-i\mathbf{k}_1 \cdot (\mathbf{x}_1 - \mathbf{x}_2)]$	$\delta(\mathbf{k}_3)$ $\times \exp[-i\mathbf{k}_1 \cdot (\mathbf{x}_1 - \mathbf{x}_2)]$	$\delta(\mathbf{k}_3)$ $\times \exp[-i\mathbf{k}_1 \cdot (\mathbf{x}_1 - \mathbf{x}_2)]$

TABLE II: momentum transfer plane wave factors for the graphs in Fig. 3.

The same for the other terms in (6.1), using the plane-wave momentum transfer factors in Table II and integrating over x_3, we arrive at the formula

$$V^{(eff)} = \rho_{NM} \int \frac{d^3k}{(2\pi)^3} e^{-i\mathbf{k}\cdot(\mathbf{x}_1-\mathbf{x}_2)} \left\{ \widetilde{V}_{12;3}(\mathbf{k}, -\mathbf{k}) + \widetilde{V}_{13;2}(\mathbf{k}, -\mathbf{k}) + \widetilde{V}_{23;1}(\mathbf{k}, -\mathbf{k}) \right\} \tag{6.10}$$

Note that because of $\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = 0$, for all three cases in Table II one has a $\mathbf{k}_3 = 0$, which is logical.

The specific momenta assignments in the three terms in (6.10) are

$$\widetilde{V}_{12;3}(\mathbf{k}, -\mathbf{k}) : \begin{cases} \mathbf{k}_1 = +\mathbf{k} \\ \mathbf{k}_2 = -\mathbf{k} \\ \mathbf{k}_3 = \mathbf{0} \end{cases}, \quad \widetilde{V}_{13;2}(\mathbf{k}, -\mathbf{k}) : \begin{cases} \mathbf{k}_1 = +\mathbf{k} \\ \mathbf{k}_2 = -\mathbf{k} \\ \mathbf{k}_3 = \mathbf{0} \end{cases}, \quad \widetilde{V}_{23:1}(\mathbf{k}, -\mathbf{k}) : \begin{cases} \mathbf{k}_1 = +\mathbf{k} \\ \mathbf{k}_2 = -\mathbf{k} \\ \mathbf{k}_3 = \mathbf{0} \end{cases}. \tag{6.11}$$

C. Potentials $\widetilde{V}_{12;3}(\mathbf{k}, -\mathbf{k}), \widetilde{V}_{13;2}(-\mathbf{k}, \mathbf{0}), \widetilde{V}_{23;1}(\mathbf{0}, +\mathbf{k})$ from Meson-Pair-Exchange

Here we give a list of the effective momentum-space potentials from meson-pair-exchange (MPE). We apply the substitution given in Eq. 6.11 to the momentum-space potentials given in section V.

(i) $J^{PC} = 0^{++}\ (\pi\pi)_0$:

$$\widetilde{V}^{(0)}_{12;3}(\mathbf{k},-\mathbf{k}) = +\left(\frac{f_P}{m_\pi}\right)^2 (\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\ (\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k})F^2_\pi(\mathbf{k}^2)\cdot \times\left\{\frac{g^{(1)}_{(\pi\pi)_0}}{m_\pi} - \frac{g^{(2)}_{(\pi\pi)_0}}{m_\pi^3}\ \mathbf{k}^2\right\}\left(\omega^2_\pi(\mathbf{k})\omega^2_\pi(\mathbf{k})\right)^{-1}\ , \tag{6.12a}$$

$$\widetilde{V}^{(0)}_{13;2}(\mathbf{k},-\mathbf{k}) = \widetilde{V}^{(0)}_{23;1}(\mathbf{k},-\mathbf{k}) = 0. \tag{6.12b}$$

Here $\omega_1 = \omega_1(\mathbf{k}_1)$ and $\omega_2 = \omega_2(\mathbf{k}_2)$, where $\mathbf{k}_1$ and $\mathbf{k}_2$ refer to the first and second argument in $\widetilde{V}_{12;3}$ respectively. This also applies to the further formulas of this subsection.

(ii) $J^{PC} = 1^{--}\ (\pi\pi)_1$: ‘

$$\widetilde{V}^{(1)}_{12;3}(\mathbf{k},-\mathbf{k}) = -\frac{g_{(\pi\pi)_1}}{m_\pi^4}\left(\frac{f_P^2}{2M}\right)(i\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k})\cdot \times\left[(\mathbf{q}_1+\mathbf{q}_2)\cdot\mathbf{k}\right]\ F^2_\pi(\mathbf{k}^2)\ \left(\omega^2_1(\mathbf{k})\omega^2_2(\mathbf{k})\right)^{-1}\ , \tag{6.13a}$$

$$\widetilde{V}^{(2)}_{12;3}(\mathbf{k},-\mathbf{k}) = 2\frac{g_{(\pi\pi)_1}}{m_\pi^4}\left(\frac{f_P^2}{2M}\right)(i\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k})\cdot \times\left[\mathbf{q}_3\cdot\mathbf{k}\right]F^2_\pi(\mathbf{k}^2)\ \left(\omega^2_1(\mathbf{k})\omega^2_2(\mathbf{k})\right)^{-1}, \tag{6.13b}$$

$$\widetilde{V}^{(1)}_{13;2}(\mathbf{k},-\mathbf{k}) = \widetilde{V}^{(1)}_{23;1}(\mathbf{k},-\mathbf{k}) = 0, \tag{6.13c}$$

$$\widetilde{V}^{(2)}_{13;2}(\mathbf{k},-\mathbf{k}) = \widetilde{V}^{(2)}_{23;1}(\mathbf{k},-\mathbf{k}) = 0\ . \tag{6.13d}$$

(iii) $J^{PC} = 1^{++}\ (\pi\rho)_1$:

$$\begin{aligned}
\widetilde{V}^{(1)}_{12;3}(\mathbf{k},-\mathbf{k}) &= +\frac{g_{(\pi\rho)_1}}{m_\pi}\left(\frac{f_P g_V}{2m_\pi M}\right)(i\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)\cdot \\
&\times\Big[2(\boldsymbol{\sigma}_1\cdot\mathbf{k})\ \boldsymbol{\sigma}_3\cdot(\mathbf{q}_3-\mathbf{q}_2)-2(\boldsymbol{\sigma}_2\cdot\mathbf{k})\ \boldsymbol{\sigma}_3\cdot(\mathbf{q}_3-\mathbf{q}_1) \\
&+i(1+\kappa_V)\Big((\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_3\cdot\mathbf{k})+(\boldsymbol{\sigma}_2\cdot\mathbf{k})(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_3\cdot\mathbf{k})\Big)\Big]\ \cdot \\
&\times F_\pi(\mathbf{k}^2)\ F_\rho(\mathbf{k}^2)\ \left(\omega_\pi^2(\mathbf{k})\ \omega_\rho^2(\mathbf{k})\right)^{-1}, \qquad (6.14a)\\
\widetilde{V}^{(1)}_{13;2}(\mathbf{k},-\mathbf{k}) &= -\frac{g_{(\pi\rho)_1}}{m_\pi}\left(\frac{f_P g_V}{2m_\pi M}\right)(i\boldsymbol{\tau}_3\times\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\cdot \\
&\times\left\{2\boldsymbol{\sigma}_1\cdot\mathbf{k}\boldsymbol{\sigma}_2\cdot(\mathbf{q}_2-\mathbf{q}_3)\right\}F_\pi(\mathbf{k}^2)\ F_\rho(\mathbf{0})\ \left(\omega_\pi^2(\mathbf{k})\ m_\rho^2\right)^{-1}, \\
\widetilde{V}^{(1)}_{23;1}(\mathbf{k},-\mathbf{k}) &= -\frac{g_{(\pi\rho)_1}}{m_\pi}\left(\frac{f_P g_V}{2m_\pi M}\right)(i\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3\cdot\boldsymbol{\tau}_1)\cdot \\
&\times\left\{2\boldsymbol{\sigma}_2\cdot\mathbf{k}\boldsymbol{\sigma}_1\cdot(\mathbf{q}_1-\mathbf{q}_3)\right\}F_\rho(\mathbf{0})\ F_\pi(\mathbf{k}^2)\ \left(\omega_\pi^2(\mathbf{k})m_\rho^2\right)^{-1}. \qquad (6.14b)
\end{aligned}$$

(iv) $J^{PC} = 1^{++}\ (\pi\sigma)_1$:

$$\begin{aligned}
\widetilde{V}^{(1)}_{12;3}(\mathbf{k},-\mathbf{k}) &= +2\frac{g_{(\pi\sigma)_1}}{m_\pi^2}\left(\frac{f_P g_S}{m_\pi}\right)\Big[(\boldsymbol{\sigma}_1\cdot\mathbf{k}\ \boldsymbol{\sigma}_3\cdot\mathbf{k})(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3)+(\boldsymbol{\sigma}_2\cdot\mathbf{k}\ \boldsymbol{\sigma}_3\cdot\mathbf{k})(\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)\Big]\cdot \\
&\times F_\pi(\mathbf{k}^2)\ F_\sigma(\mathbf{k}^2)\ \left(\omega_\pi^2\ \omega_\sigma^2\right)^{-1}, \qquad (6.15a)\\
\widetilde{V}^{(2)}_{12;3}(\mathbf{k},-\mathbf{k}) &= -4\frac{g_{(\pi\sigma)_1}}{m_\pi^2}\left(\frac{f_P g_S}{2m_\pi M^2}\right)\Big[(\boldsymbol{\sigma}_1\cdot\mathbf{k}\ \boldsymbol{\sigma}_3\cdot\mathbf{q}_3)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3)-(\boldsymbol{\sigma}_2\cdot\mathbf{k}\ \boldsymbol{\sigma}_3\cdot\mathbf{q}_3)(\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)\Big]\cdot \\
&\times\Big[(\mathbf{q}_1+\mathbf{q}_2)\cdot\mathbf{k}+\mathbf{k}^2\Big]\ F_\pi(\mathbf{k}^2)\ F_\sigma(\mathbf{k}^2)\ \left(\omega_\pi^2\ \omega_\sigma^2\right)^{-1}, \qquad (6.15b)\\
\widetilde{V}^{(1)}_{13;2}(\mathbf{k},-\mathbf{k}) &= +\frac{g_{(\pi\sigma)_1}}{m_\pi^2}\left(\frac{f_P g_S}{m_\pi}\right)\Big[(\boldsymbol{\sigma}_1\cdot\mathbf{k}\ \boldsymbol{\sigma}_2\cdot\mathbf{k})(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\ \left(\omega_\pi^2(\mathbf{k})\ m_\sigma^2\right)^{-1}\Big]\cdot \\
&\times F_\pi(\mathbf{k}^2)\ F_\sigma(\mathbf{0}), \qquad (6.15c)\\
\widetilde{V}^{(2)}_{13;2}(\mathbf{k},-\mathbf{k}) &= -2\frac{g_{(\pi\sigma)_1}}{m_\pi^2}\left(\frac{f_P g_S}{2m_\pi M^2}\right)\Big[(\boldsymbol{\sigma}_1\cdot\mathbf{k}\ \boldsymbol{\sigma}_2\cdot\mathbf{q}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\omega_\pi^2(\mathbf{k})\ m_\sigma^2\right)^{-1}\Big]\cdot \\
&\times\Big[2\mathbf{q}_1\cdot\mathbf{k}_1+\mathbf{k}_1^2\Big]\ F_\pi(\mathbf{k}^2)\ F_\sigma(\mathbf{0}), \qquad (6.15d)\\
\widetilde{V}^{(1)}_{23;1}(\mathbf{k},-\mathbf{k}) &= +\frac{g_{(\pi\sigma)_1}}{m_\pi^2}\left(\frac{f_P g_S}{m_\pi}\right)\Big[(\boldsymbol{\sigma}_1\cdot\mathbf{k}\ \boldsymbol{\sigma}_2\cdot\mathbf{k})(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\ \left(\omega_\pi^2(\mathbf{k})\ m_\sigma^2\right)^{-1}\Big]\cdot \\
&\times F_\pi(\mathbf{k}^2)\ F_\sigma(\mathbf{k}^2), \qquad (6.15e)\\
\widetilde{V}^{(2)}_{23;1}(\mathbf{k},-\mathbf{k}) &= -2\frac{g_{(\pi\sigma)_1}}{m_\pi^2}\left(\frac{f_P g_S}{2m_\pi M^2}\right)\Big[(\boldsymbol{\sigma}_2\cdot\mathbf{k}\ \boldsymbol{\sigma}_1\cdot\mathbf{q}_1)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\omega_\pi^2(\mathbf{k}\ m_\sigma^2)\right)^{-1}\Big]\cdot \\
&\times\Big[2\mathbf{q}_2\cdot\mathbf{k}_2+\mathbf{k}_2^2\Big]\ F_\pi(\mathbf{k}^2)\ F_\sigma(\mathbf{0}). \qquad (6.15f)
\end{aligned}$$

(v) $J^{PC} = 1^{+-}$ $(\pi\omega)_1$:

$$\begin{aligned}
\widetilde{V}^{(1)}_{12;3}(\mathbf{k},-\mathbf{k}) &= +2\frac{g_{(\pi\omega)_1}}{m_\pi^2}\left(\frac{f_P g_V}{2m_\pi M^2}\right)\left[\left(2\boldsymbol{\sigma}_3\cdot\mathbf{q}_1 - i(1+\kappa_V)\,\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_3\cdot\mathbf{k}\right)(\boldsymbol{\sigma}_2\cdot\mathbf{k})\,(\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)\right.\\
&\quad \left. -\,(2\boldsymbol{\sigma}_3\cdot\mathbf{q}_2 + i(1+\kappa_V)\,\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_3\cdot\mathbf{k})\,(\boldsymbol{\sigma}_1\cdot\mathbf{k})\,(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3)\right]\cdot\\
&\quad \times\left[(\mathbf{p}_1-\mathbf{p}_2)\cdot\mathbf{k}+\mathbf{k}^2\right]\,F_\pi(\mathbf{k}^2)\,F_\omega(\mathbf{k}^2)\,\left(\omega_\pi^2(\mathbf{k})\,\omega_\omega(\mathbf{k})^2\right)^{-1}, \qquad (6.16a)\\
\widetilde{V}^{(2)}_{12;3}(\mathbf{k},-\mathbf{k}) &= 0, \qquad (6.16b)\\
\widetilde{V}^{(1)}_{13;2}(\mathbf{k},-\mathbf{k}) &= -2\frac{g_{(\pi\omega)_1}}{m_\pi^2}\left(\frac{f_P g_V}{2m_\pi M^2}\right)\left[(2\boldsymbol{\sigma}_2\cdot\mathbf{q}_3)\,(\boldsymbol{\sigma}_1\cdot\mathbf{k})\right]\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left(\omega_\pi^2(\mathbf{k}\;m_\omega^2)\right)^{-1}\cdot\\
&\quad \times\left[(2\mathbf{p}_1\cdot\mathbf{k}_1)+\mathbf{k}_1^2\right]F_\pi(\mathbf{k}^2)\,F_\omega(\mathbf{0}), \qquad (6.16c)\\
\widetilde{V}^{(2)}_{13;2}(\mathbf{k},-\mathbf{k}) &= -\frac{g_{(\pi\omega)_1}}{m_\pi^2}\left(\frac{f_P g_V}{m_\pi}\right)(\boldsymbol{\sigma}_1\cdot\mathbf{k})\,(\boldsymbol{\sigma}_2\cdot\mathbf{k})\,(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\,\left(\omega_\pi^2(\mathbf{k})\,m_\omega^2\right)^{-1}\cdot\\
&\quad \times F_\pi(\mathbf{k}^2)\,F_\omega(\mathbf{0}), \qquad (6.16d)\\
\widetilde{V}^{(1)}_{23;1}(\mathbf{k},-\mathbf{k}) &= +\frac{g_{(\pi\omega)_1}}{m_\pi^2}\left(\frac{f_P g_V}{2m_\pi M^2}\right)\left[(2\boldsymbol{\sigma}_1\cdot\mathbf{q}_3)\,(\boldsymbol{\sigma}_2\cdot\mathbf{k})\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right]\,\left(\omega_\pi^2(\mathbf{k})\,m_\omega^2\right)^{-1}\cdot\\
&\quad \times\left[(2\mathbf{p}_2\cdot\mathbf{k}_2+\mathbf{k}_2^2)\right]F_\pi(\mathbf{k}^2)\,F_\omega(\mathbf{0}), \qquad (6.16e)\\
\widetilde{V}^{(2)}_{23;1}(\mathbf{k},-\mathbf{k}) &= -\frac{g_{(\pi\omega)_1}}{m_\pi^2}\left(\frac{f_P g_V}{m_\pi}\right)(\boldsymbol{\sigma}_1\cdot\mathbf{k})\,(\boldsymbol{\sigma}_2\cdot\mathbf{k})\,(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\,\left(\omega_\pi^2(\mathbf{k})\,m_\omega^2\right)^{-1}\cdot\\
&\quad \times F_\pi(\mathbf{k}^2)\,F_\omega(\mathbf{0}). \qquad (6.16f)
\end{aligned}$$

VII. LNR-APPROXIMATION TWO-MESON-PAIR EXCHANGE POTENTIALS IN MOMENTUM-SPACE

The LNR-approximation [21] leads to a massive reduction of the number of terms in the two-meson-pair exchange potentials given in section V. For example, the $(\pi\pi)_1$- and $(\pi\rho)_1$-potentials vanish completely. In this section we give the reduced form of these MPE-potentials after applying the Tr-operation: $Tr\boldsymbol{\tau}_3 = Tr\boldsymbol{\sigma}_3 = 0$.

(i) $J^{PC} = 0^{++}\ (\pi\pi)_0$:

$$\widetilde{V}^{(0)}_{12;3}(\mathbf{k},-\mathbf{k}) = +\left(\frac{f_P}{m_\pi}\right)^2 (\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\ (\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k})F^2_\pi(\mathbf{k}^2)\cdot \times\left\{\frac{g^{(1)}_{(\pi\pi)_0}}{m_\pi} - \frac{g^{(2)}_{(\pi\pi)_0}}{m^3_\pi}\ \mathbf{k}^2\right\}\left(\omega^2_\pi(\mathbf{k})\omega^2_\pi(\mathbf{k})\right)^{-1}\ , \tag{7.1a}$$

$$\widetilde{V}^{(0)}_{13;2}(\mathbf{k},-\mathbf{k}) = \widetilde{V}^{(0)}_{23;1}(\mathbf{k},-\mathbf{k}) = 0. \tag{7.1b}$$

Here $\omega_1 = \omega_1(\mathbf{k}_1)$ and $\omega_2 = \omega_2(\mathbf{k}_2)$, where $\mathbf{k}_1$ and $\mathbf{k}_2$ refer to the first and second argument in $\widetilde{V}_{12;3}$ respectively.
Remark: the $(\eta_8\eta_8)$-potential is obtained from (7.4e) by the substitutions $m_\pi \to m_\eta$ and $(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) \to 1$.

(ii) $JPC = 1^{--}\ (\pi\pi)_1$: ‘

$$\widetilde{V}^{(1)}_{12;3}(\mathbf{k},-\mathbf{k}) = 0,\ \ \widetilde{V}^{(2)}_{12;3}(\mathbf{k},-\mathbf{k}) = 0, \tag{7.2a}$$

$$\widetilde{V}^{(1)}_{13;2}(\mathbf{k},-\mathbf{k}) = \widetilde{V}^{(1)}_{23;1}(\mathbf{k},-\mathbf{k}) = 0, \tag{7.2b}$$

$$\widetilde{V}^{(2)}_{13;2}(\mathbf{k},-\mathbf{k}) = \widetilde{V}^{(2)}_{23;1}(\mathbf{k},-\mathbf{k}) = 0\ . \tag{7.2c}$$

Remark: (1) For NN there are no other pairs with the same quantum numbers and having $I = 0$, see Appendix G; (2) Interaction term $(i\sqrt{3}/2)\ \phi^\mu\left(K^\dagger\ \overset{\leftrightarrow}{\partial}_\mu\ K\right)$ gives for $V_{12;3}$ terms $\propto\ \mathbf{q}_3 \to 0$ and $\propto\ (\mathbf{q}_1+\mathbf{q}_2) \to 0$.

(iii) $JPC = 1^{++}\ (\pi\rho)_1$:

$$\widetilde{V}^{(1)}_{12;3}(\mathbf{k},-\mathbf{k}) = 0,\ \ \widetilde{V}^{(1)}_{13;2}(-\mathbf{k},\mathbf{0}) = 0,\ \ \widetilde{V}^{(1)}_{23;1}(-\mathbf{k},\mathbf{0}) = 0. \tag{7.3}$$

Remark: (1) For NN there are no other pairs with the same quantum numbers and having $I = 0$, see Appendix G, (2) Interaction $-i(\sqrt{3}/2)(K^\dagger\cdot K_A)\phi_8$ gives term $\boldsymbol{\sigma}_1\cdot\mathbf{k}\ \boldsymbol{\sigma}_2\cdot\mathbf{q} - \boldsymbol{\sigma}_1\cdot\mathbf{q}\ \boldsymbol{\sigma}_2\cdot\mathbf{k}$ whose Fourier transform vanishes, (3) The interaction $i\sqrt{3}/2)\ f_1\left(K^\dagger\cdot K^* - K^{*\dagger}\cdot K\right)$ gives for $V_{12;3}$ terms $\propto \boldsymbol{\sigma}_3 \to 0$.

(iv) $J^{PC} = 1^{++}$ $(\pi\sigma)_1$:

$$
\begin{aligned}
\widetilde{V}^{(1)}_{12;3}(\mathbf{k},-\mathbf{k}) &= 0, \quad \widetilde{V}^{(2)}_{12;3}(\mathbf{k},-\mathbf{k}) = 0, & \text{(7.4a)}\\
\widetilde{V}^{(1)}_{13;2}(\mathbf{k},-\mathbf{k}) &= +\frac{g_{(\pi\sigma)_1}}{m_\pi^2}\left(\frac{f_P g_S}{m_\pi}\right)\left[(\boldsymbol{\sigma}_1\cdot\mathbf{k}\ \boldsymbol{\sigma}_2\cdot\mathbf{k})(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\ \left(\omega_\pi^2(\mathbf{k})\ m_\sigma^2\right)^{-1}\right]\cdot \\
&\quad \times F_\pi(\mathbf{k}^2)\ F_\sigma(\mathbf{0}), & \text{(7.4b)}\\
\widetilde{V}^{(2)}_{13;2}(\mathbf{k},-\mathbf{k}) &= -2\frac{g_{(\pi\sigma)_1}}{m_\pi^2}\left(\frac{f_P g_S}{2m_\pi M^2}\right)\left[(\boldsymbol{\sigma}_1\cdot\mathbf{k}\ \boldsymbol{\sigma}_2\cdot\mathbf{q}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\omega_\pi^2(\mathbf{k})\ m_\sigma^2\right)^{-1}\right]\cdot \\
&\quad \times\left[2\mathbf{q}_1\cdot\mathbf{k}_1+\mathbf{k}_1^2\right]\ F_\pi(\mathbf{k}^2)\ F_\sigma(\mathbf{0}), & \text{(7.4c)}\\
\widetilde{V}^{(1)}_{23;1}(\mathbf{k},-\mathbf{k}) &= +\frac{g_{(\pi\sigma)_1}}{m_\pi^2}\left(\frac{f_P g_S}{m_\pi}\right)\left[(\boldsymbol{\sigma}_1\cdot\mathbf{k}\ \boldsymbol{\sigma}_2\cdot\mathbf{k})(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\ \left(\omega_\pi^2(\mathbf{k})\ m_\sigma^2\right)^{-1}\right]\cdot \\
&\quad \times F_\pi(\mathbf{k}^2)\ F_\sigma(\mathbf{0}), & \text{(7.4d)}\\
\widetilde{V}^{(2)}_{23;1}(\mathbf{k},-\mathbf{k}) &= -2\frac{g_{(\pi\sigma)_1}}{m_\pi^2}\left(\frac{f_P g_S}{2m_\pi M^2}\right)\left[(\boldsymbol{\sigma}_2\cdot\mathbf{k}\ \boldsymbol{\sigma}_1\cdot\mathbf{q}_1)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\omega_\pi^2(\mathbf{k})\ m_\sigma^2\right)^{-1}\right]\cdot \\
&\quad \times\left[2\mathbf{q}_2\cdot\mathbf{k}_2+\mathbf{k}_2^2\right]\ F_\pi(\mathbf{k}^2)\ F_\sigma(\mathbf{0}). & \text{(7.4e)}
\end{aligned}
$$

Remark 1: the $(\eta_8\sigma)_0$-potential is obtained from (7.4e) by the substitutions $m_\pi \to m_\eta$ and $(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) \to 1$.

Remark 2: as discussed in section VI A (a) in the two-particle CM-system $\mathbf{q}_1 = \mathbf{q}_2 \equiv \mathbf{q}$, and (b) averaging gives $\mathbf{p}_3 = \mathbf{q}_3 = 0$. Therefore, we have

$$
\begin{aligned}
(\boldsymbol{\sigma}_2\cdot\mathbf{q}_2)(\mathbf{q}_1+\mathbf{q}_3)\cdot\mathbf{k} &\to -(\boldsymbol{\sigma}_2\cdot\mathbf{q})(\mathbf{q}\cdot\mathbf{k})\\
(\boldsymbol{\sigma}_1\cdot\mathbf{q}_1)(\mathbf{q}_2+\mathbf{q}_3)\cdot\mathbf{k} &\to -(\boldsymbol{\sigma}_1\cdot\mathbf{q})(\mathbf{q}\cdot\mathbf{k}). \qquad \text{(7.5)}
\end{aligned}
$$

The final step to the configuration-space potentials is now standard and straightforward, see Appendix B.

(v) $J^{PC} = 1^{+-}$ $(\pi\omega)_1$:

$$\widetilde{V}^{(1)}_{12;3}(\mathbf{k},-\mathbf{k}) \;=\; 0,\;\; \widetilde{V}^{(2)}_{12;3}(\mathbf{k},-\mathbf{k}) = 0\;, \tag{7.6a}$$

$$\widetilde{V}^{(1)}_{13;2}(\mathbf{k},-\mathbf{k}) \;=\; -4\frac{g_{(\pi\omega)_1}}{m_\pi^2}\left(\frac{f_P g_V}{2m_\pi M^2}\right)\left[(\boldsymbol{\sigma}_2\cdot\mathbf{q}_3)\,(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\omega_\pi^2(\mathbf{k})\;m_\omega^2\right)^{-1}\right]\cdot \times\left[2\mathbf{p}_1\cdot\mathbf{k}+\mathbf{k}^2\right]F_\pi(\mathbf{k}^2)\;F_\omega(\mathbf{0}), \tag{7.6b}$$

$$\widetilde{V}^{(2)}_{13;2}(\mathbf{k},-\mathbf{k}) \;=\; -\frac{g_{(\pi\omega)_1}}{m_\pi^2}\left(\frac{f_P g_V}{2m_\pi}\right)\left[(\boldsymbol{\sigma}_1\cdot\mathbf{k})\,(\boldsymbol{\sigma}_2\cdot\mathbf{k})\,(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\omega_\pi^2(\mathbf{k})\;m_\omega^2\right)^{-1}\right]\cdot \times F_\pi(\mathbf{k}^2)\;F_\omega(\mathbf{0}), \tag{7.6c}$$

$$\widetilde{V}^{(1)}_{23;1}(\mathbf{k},-\mathbf{k}) \;=\; +2\frac{g_{(\pi\omega)_1}}{m_\pi^2}\left(\frac{f_P g_V}{2m_\pi M^2}\right)\left[(\boldsymbol{\sigma}_1\cdot\mathbf{q}_3)\,(\boldsymbol{\sigma}_2\cdot\mathbf{k})(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\omega_\pi^2(\mathbf{k})\;m_\omega^2\right)^{-1}\right]\cdot \times\left[2\mathbf{p}_2\cdot\mathbf{k}_2+\mathbf{k}_2^2\right]F_\pi(\mathbf{k}^2)\;F_\omega(\mathbf{0}), \tag{7.6d}$$

$$\widetilde{V}^{(2)}_{23;1}(\mathbf{k},-\mathbf{k}) \;=\; -\frac{g_{(\pi\omega)_1}}{m_\pi^2}\left(\frac{f_P g_V}{2m_\pi}\right)\left[(\boldsymbol{\sigma}_1\cdot\mathbf{k})\,(\boldsymbol{\sigma}_2\cdot\mathbf{k})\,(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\omega_\pi^2(\mathbf{k})\;m_\omega^2\right)^{-1}\right]\cdot \times F_\pi(\mathbf{k}^2)\;F_\omega(\mathbf{0}). \tag{7.6e}$$

Remark 1: the $(\eta_8\phi_8)_0$-potential is obtained from (7.6e) by the substitutions $m_\pi \to m_\eta, m_\omega \to m_\phi$ and $(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\to 1$.
Remark 2: as discussed in section VI A (a) (a) $\mathbf{p}_3 = \mathbf{q}_3 = 0$, and (b) $\mathbf{q}_{3,i}\mathbf{p}_{3,j} \to \frac{1}{5}k_F^2\;\delta_{ij}$ in nuclear matter. So, in (7.4e) the linear terms in either $\mathbf{q}_3$ or $\mathbf{p}_3$ vanish and for the quadratic terms

$$(\boldsymbol{\sigma}_{1,2}\cdot\mathbf{q}_3)(\mathbf{p}_3\cdot\mathbf{k}) \to \frac{1}{5}k_F^2\;(\boldsymbol{\sigma}_{1,2}\cdot\mathbf{k}). \tag{7.7}$$

With these proviso's the derivation of the configuration potential is straightforward, see Appendix B.

(vi) $J^{PC} = 0^{++}$ $(\sigma\sigma)$-Pair Exchange Potential:

$$\widetilde{V}^{0}_{12;3}(\mathbf{k},-\mathbf{k}) = -\frac{g_{(\sigma\sigma)}}{m_\pi}\;g_S^2\;F_\sigma^2(\mathbf{k}^2)\;\left(\omega_\sigma^2\omega_\sigma^2\right)^{-1}, \tag{7.8a}$$

$$\widetilde{V}^{0}_{13;2}(\mathbf{k},-\mathbf{k}) = -\frac{g_{(\sigma\sigma)}}{m_\pi}\;g_S^2\;F_\sigma(\mathbf{k}^2)\;F_\sigma(\mathbf{0})\;\left(\omega_\sigma^2\;m_\sigma^2\right)^{-1}, \tag{7.8b}$$

$$\widetilde{V}^{0}_{23;1}(\mathbf{k},-\mathbf{k}) = \widetilde{V}^{0}_{13;2}(\mathbf{k},-\mathbf{k}). \tag{7.8c}$$

VIII. CONFIGURATION-SPACE THREE-BODY INDUCED HYPERON-NUCLEON POTENTIALS

The so-called configuration-space *effective* nucleon-nucleon potentials from the previous section VII are recorded below. In this part only the dominating terms are included, i.e. we neglect the terms proportional to $1/M^2$. For the different J^{PC}-types we obtain:

---

ad (i) $J^{PC} = 0^{++}$ $(\pi\pi)_0$-Pair Exchange Potential: From (7.1b) one gets

$$\begin{aligned}
V^{(1)}_{(\pi\pi)_0}(r) &= -\frac{(4\pi\rho_{NM})}{m_\pi^3}\frac{g^{(1)}_{(\pi\pi)_0}}{4\pi}\frac{f^2}{4\pi}\cdot\left(\frac{m_\pi}{m_{\pi^+}}\right)^2\cdot(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\ (\boldsymbol{\sigma}_1\cdot\boldsymbol{\nabla}\ \boldsymbol{\sigma}_2\cdot\boldsymbol{\nabla})\ \cdot \\
&\quad\times\frac{1}{2m_\pi}\frac{d}{dm_\pi}\left[m_\pi\ \phi^0_C\left(m_\pi,\frac{\Lambda_\pi}{\sqrt{2}},r\right)\right] \\
&= -\frac{(4\pi\rho_{NM})}{m_\pi^3}\frac{g^{(1)}_{(\pi\pi)_0}}{4\pi}\frac{f^2}{4\pi}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\ \cdot\left(\frac{m_\pi}{m_{\pi^+}}\right)^2\cdot\left(\frac{1}{2m_\pi}\frac{d}{dm_\pi}\right) \\
&\quad\times\left[m_\pi^3\cdot\left\{\frac{1}{3}\phi^1_C\left(m_\pi,\frac{\Lambda_\pi}{\sqrt{2}},r\right)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)+\phi^0_T\left(m_\pi,\frac{\Lambda_\pi}{\sqrt{2}},r\right)\ S_{12}\right\}\right] \\
&= -\frac{(4\pi\rho_{NM})}{m_\pi^3}\frac{g^{(1)}_{(\pi\pi)_0}}{4\pi}\frac{f^2}{4\pi}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\ \cdot\left(\frac{m_\pi}{m_{\pi^+}}\right)^2\cdot m_\pi\ \cdot \\
&\quad\times\left[\frac{1}{3}\psi^1_C\left(m_\pi,\frac{\Lambda_\pi}{\sqrt{2}},r\right)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)+\psi^0_T\left(m_\pi,\frac{\Lambda_\pi}{\sqrt{2}},r\right)\ S_{12}\right],
\end{aligned}\tag{8.1}$$

where the functions $\psi^0_C(m,\Lambda,2)$ etc. are defined in Appendix D.

An alternative way for the presentation of this potential is as follows: Using

$$(\omega^2\omega^2)^{-1} = \lim_{m'\to m}\left[\frac{1}{\mathbf{k}^2+m^2}-\frac{1}{\mathbf{k}^2+m'^2}\right]/(m'^2-m^2),$$

we have from (7.1b)

$$\begin{aligned}
V_{(\pi\pi)_0}(r) &= \lim_{m'\to m}\ \frac{(4\pi\rho_{NM})}{m_\pi^3}\frac{g^{(1)}_{(\pi\pi)_0}}{4\pi}\frac{f^2}{4\pi}\cdot\left(\frac{m_\pi}{m_{\pi^+}}\right)^2\cdot(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\ (\boldsymbol{\sigma}_1\cdot\boldsymbol{\nabla}\ \boldsymbol{\sigma}_2\cdot\boldsymbol{\nabla})\ \cdot \\
&\quad\times\left[m\ \phi^0_C(m,\frac{\Lambda_\pi}{\sqrt{2}},r)-m'\ \phi^0_C(m',\frac{\Lambda_\pi}{\sqrt{2}},r)\right]/(m'^2-m^2),
\end{aligned}\tag{8.2}$$

Working out the differentiations and defining $\widetilde{\phi}^0_C = (m/m_\pi)\ \phi^0_C$ we get

$$\begin{aligned}
V_{(\pi\pi)_0}(r) &= \lim_{m'\to m}\ \frac{(4\pi\rho_{NM})}{m_\pi^3}\frac{g^{(1)}_{(\pi\pi)_0}}{4\pi}\frac{f^2}{4\pi}\cdot\left(\frac{m_\pi}{m_{\pi^+}}\right)^2\cdot(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\ \cdot \\
&\quad\times\left[\left\{\frac{1}{3}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\widetilde{\phi}^1_C(m,r)+S_{12}\ \widetilde{\phi}^0_T(m,r)\right\}\right. \\
&\quad\left.-\left\{\frac{1}{3}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\widetilde{\phi}^1_C(m',r)+S_{12}\ \widetilde{\phi}^0_T(m',r)\right\}\right]\ \frac{m_\pi^3}{m'^2-m^2},
\end{aligned}\tag{8.3}$$

where

$$\widetilde{\phi}^1_C(m,r) \;=\; \left(\frac{d^2}{dx^2}+\frac{2}{x}\frac{d}{dx}\right)\;\widetilde{\phi}^0_C(m,r),\;\widetilde{\phi}^0_T(m,r)=\frac{1}{3}\left(\frac{d^2}{dx^2}-\frac{1}{x}\frac{d}{dx}\right)\;\widetilde{\phi}^0_C(m,r).$$

Using in (7.1b) the relation $\mathbf{k}^2=\omega^2(\mathbf{k})-m_\pi^2$, the derivative interaction gives two contributions $V^{(2a)}_{(\pi\pi)_0}$ and $V^{(2b)}_{(\pi\pi)_0}$. $V^{(2a)}_{(\pi\pi)_0}$ is obtained from $V^{(1)}_{(\pi\pi)_0}$ with the substitution $g^{(1)}_{(\pi\pi)_0}\to -g^{(2)}_{(\pi\pi)_0}$. The other contribution is

$$\begin{aligned}V^{(2b)}_{(\pi\pi)_0}(r) \;=\; &+2\frac{(4\pi\rho_{NM})}{m_\pi^3}\frac{g^{(2)}_{(\pi\pi)_0}}{4\pi}\;\frac{f^2}{4\pi}\cdot\left(\frac{m_\pi}{m_{\pi^+}}\right)^2\cdot(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\cdot\\ &\times\left[\frac{1}{3}\phi^1_C(m_\pi,\Lambda_\pi,r)\;\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+\phi^0_T(m_\pi,\Lambda_\pi,r)\right].\end{aligned} \tag{8.4}$$

The weights of the graphs Fig. 2 are: $w(12;3)=1, w(13;2)=w(23;1)=0$

Similarly potentials for the $(\eta_8\eta_8)$-pair: $f\to f_\eta, m_\pi\to m_\eta$, and $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\to 1$. We neglect here the mixing angle θ_P, i.e. $\eta_8\approx\eta(548)$.

---

ad (iv) $J^{PC}=1^{++}$ $(\pi\sigma)_1$-Pair Exchange Potential:

$$\begin{aligned}V^{(1)}_{(\pi\sigma)_1}(r) \;=\; &-2\frac{(4\pi\rho_{NM})}{m_\pi^3}\;\frac{g_{(\pi\sigma)_1}}{4\pi}\frac{f_P g_S}{4\pi}\,(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\;\;(\boldsymbol{\sigma}_1\cdot\boldsymbol{\nabla})(\boldsymbol{\sigma}_2\cdot\boldsymbol{\nabla})\;\cdot\\ &\times\left[m_\pi\;\phi^0_C(m_\pi,\Lambda_\pi,r)\right]\cdot\left(m_\sigma^2\right)^{-1}\\ =\; &-2\frac{(4\pi\rho_{NM})}{m_\pi^3}\;\frac{g_{(\pi\sigma)_1}}{4\pi}\frac{f_P g_S}{4\pi}\,(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\;\left(\frac{m_\pi^3}{m_\sigma^2}\right)\cdot\\ &\times\left[\frac{1}{3}\phi^1_C(m_\pi,\Lambda_\pi,r)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)+\phi^0_T(m_\pi,\Lambda_\pi,r)\;S_{12}\right],\end{aligned} \tag{8.5a}$$

$$\begin{aligned}V^{(2)}_{(\pi\sigma)_1}(r) \;=\; &-\frac{1}{2}\frac{(4\pi\rho_{NM})}{m_\pi^3}\;\frac{g_{(\pi\sigma)_1}}{4\pi}\frac{f_P g_S}{4\pi}\frac{m_\pi^2}{M^2}\;(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\;\;(\boldsymbol{\sigma}_1\cdot\boldsymbol{\nabla})(\boldsymbol{\sigma}_2\cdot\boldsymbol{\nabla})\;\cdot\\ &\times\left[m_\pi^3\;\phi^1_C(m_\pi,\Lambda_\pi,r)\right]\cdot\left(m_\sigma^2\right)^{-1}\\ =\; &-\frac{1}{2}\frac{(4\pi\rho_{NM})}{m_\pi^3}\;\frac{g_{(\pi\sigma)_1}}{4\pi}\frac{f_P g_S}{4\pi}\frac{m_\pi^2}{M^2}\;(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\;\left(\frac{m_\pi^3}{m_\sigma^2}\right)\cdot\\ &\times\left[\frac{1}{3}\phi^2_C(m_\pi,\Lambda_\pi,r)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)+\phi^1_T(m_\pi,\Lambda_\pi,r)\;S_{12}\right],\end{aligned} \tag{8.5b}$$

where in the $V^{(2)}_{(\pi\sigma)_1}$-potential we only included the local contribution (see Appendix B for more details).

The weights of the graphs Fig. 2 are: $w^{(1,2)}(12;3)=0, w^{(1)}(13;2)=w^{(1)}(23;1)=1/2$ and $w^{(2)}(13;2)=w^{(2)}(23;1)=1/2$.

Similarly for the $(\eta_8\sigma)$-pair: $f\to f_\eta, m_\pi\to m_\eta$, and $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\to 1$.

(v) $J^{PC}=1^{+-}$ $(\pi\omega)_1$-Pair Exchange Potential:

$$\begin{aligned}
V^{(2)}_{(\pi\omega)_1}(r) &= +\frac{(4\pi\rho_{NM})}{m_\pi^3}\ \frac{g_{(\pi\omega)_1}}{4\pi}\frac{f_P g_V\sin(\theta_V)}{4\pi}\,(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\ \ (\boldsymbol{\sigma}_1\cdot\boldsymbol{\nabla})(\boldsymbol{\sigma}_2\cdot\boldsymbol{\nabla})\ \cdot \\
&\quad \times\left[m_\pi\ \phi^0_C(m_\pi,\Lambda_\pi,r)\right]\cdot\left(m_\omega^2\right)^{-1} \\
&= +\frac{(4\pi\rho_{NM})}{m_\pi^3}\ \frac{g_{(\pi\omega)_1}}{4\pi}\frac{f_P g_V\sin(\theta_V)}{4\pi}\,(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\ \left(\frac{m_\pi^3}{m_\omega^2}\right)\cdot \\
&\quad \times\left[\frac{1}{3}\phi^1_C(m_\pi,\Lambda_\pi,r)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)+\phi^0_T(m_\pi,\Lambda_\pi,r)\ S_{12}\right\}. \qquad (8.6)
\end{aligned}$$

The weights of the graphs Fig. 2 are: $w(12;3)=0, w(13;2)=w(23;1)=1/2$.

Similarly for (a) $(\eta_8\phi_8)$-pair: $f\to f_\eta, m_\pi\to m_\eta$, $g_{(\pi\omega)}\sin(\theta_V)\to -g_{(\pi\omega)}/2$, $g_V\sin(\theta)\to g_V\cos(\theta_V)$ and $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\to 1$, for (b) $(\pi\rho)_0$-pair: $g_V\sin(\theta_V)\to g_V, m_\omega\to m_\rho$, $g_{(\pi\omega)}\to +g_{(\pi\omega)}/2$.

(vi) $J^{PC}=0^{++}$ $(\sigma\sigma)$-Pair Exchange Potential: For this simplest case, i.e. $(\sigma\sigma)$-pair (5.13), we get

$$\begin{aligned}
V^{(eff)}_{(\sigma\sigma)} &= -\rho_{NM}\frac{g_{(\sigma\sigma)}}{m_\pi}\ g_S^2\left[-\frac{d}{2m_\sigma dm_\sigma}I_2\left(r_{12},m_\sigma,\Lambda/\sqrt{2}\right)+2m_\sigma^{-1}\ \phi^0_C(m_\sigma,\Lambda_\sigma,r)\right] \\
&= -2\frac{(4\pi\rho_{NM})}{m_\pi^3}\frac{g_{(\sigma\sigma)}}{4\pi}\frac{g_S^2}{4\pi}\left(\frac{m_\pi^2}{m_\sigma}\right)\left[\phi^0_C(m_\sigma,\Lambda_\sigma,r)-\frac{m_\sigma^2}{\Lambda_\sigma^2}\ \psi^0_C(m_\sigma,\Lambda_\sigma/\sqrt{2},r)\right]. \quad (8.7)
\end{aligned}$$

The weights of the graphs Fig. 2 are: $w(12;3)=w(13;2)=w(23;1)=1/3$.

In the calculations we can use the explicit d/dm_σ differentiation as above, or the formula

$$(\omega^2\omega^2)^{-1}=\lim_{m'\to m}\left[\frac{1}{\mathbf{k}^2+m^2}-\frac{1}{\mathbf{k}^2+m'^2}\right]/(m'^2-m^2),$$

This gives, using again $\widetilde{\phi}^0_C=(m/m_\pi)\phi^0_C$,

$$\begin{aligned}
V^{(eff)}_{(\sigma\sigma)} &= -\frac{4\pi\rho_{NM}}{m_\pi}\frac{g_{(\sigma\sigma)}}{4\pi}\ \frac{g_S^2}{4\pi}\left[\lim_{m'_\sigma\to m_\sigma}\left\{\frac{m_\sigma\phi^0_C(m_\sigma,\Lambda_\sigma,r)-m'_\sigma\phi^0_C(m'_\sigma,\Lambda_\sigma,r)}{(m'^2_\sigma-m^2_\sigma)}\right\}\right. \\
&\qquad\qquad \left.+2m_\sigma^{-1}\ \phi^0_C(m_\sigma,\Lambda_\sigma,r)\right] \\
&= -\left(\frac{4\pi\rho_{NM}}{m_\pi^3}\right)\frac{g_{(\sigma\sigma)}}{4\pi}\ \frac{g_S^2}{4\pi}\left[+2\frac{m_\pi^2}{m_\sigma^2}\ \widetilde{\phi}^0_C(m_\sigma,\Lambda_\sigma,r)+\right. \\
&\quad \left.+\lim_{m'_\sigma\to m_\sigma}\frac{m_\pi^2}{(m'^2_\sigma-m^2_\sigma)}\left\{\widetilde{\phi}^0_C(m_\sigma,\Lambda_\sigma,r)-\widetilde{\phi}^0_C(m'_\sigma,\Lambda_\sigma,r)\right\}\right]\cdot m_\pi. \qquad (8.8)
\end{aligned}$$

TABLE III: ESC08c (rationalized) coupling constants, $F/(F+D)$-ratio's, mixing angles etc. The values with $\star$) have been determined in the fit to the YN-data. The other parameters are theoretical input or determined by the fitted parameters and the constraint from the NN-analysis. Parameter set: parbbsc.marius17a.

mesons		{1}	{8}	$F/(F+D)$	angles
ps-scalar	f	0.2926	0.2686	$\alpha_P = 0.365$	$\theta_P = -13.00^{0}$ $^{*)}$
vector	g	3.4100	0.6908	$\alpha_V^e = 1.0^{*)}$	$\theta_V = 37.50^{0}$ $^{*)}$
	f	–2.5085	3.8840	$\alpha_V^m = 0.475^{*)}$	
axial(A)	g	–0.9633	–0.8289	$\alpha_A = 0.372$	$\theta_A = +50.0^{0}$ $^{*)}$
	f	–2.8750	–2.5470	$\alpha_A^p = 0.372^{*)}$	
axial(B)	f	–0.1027	–0.2054	$\alpha_B = 0.40^{*)}$	$\theta_B = 35.26^{0}$ $^{*)}$
scalar	g	4.1821	0.6130	$\alpha_S = 1.00$	$\theta_S = 37.26^{0}$ $^{*)}$
diffractive	g_P	3.3581	0.0000	$\alpha_D =$ —	$a_{PB} = 0.25^{*)}$
	g_O	4.484	0.0000	$\alpha_D =$ —	$\psi_D = 0.0^{0}$ $^{*)}$
	f_O	–4.323	0.0000	$\alpha_D =$ —	

IX. MULTI-POMERON EFFECTIVE 2-BODY POTENTIALS

The multi-pomeron potential (MPP) we tune to the MPb model used in [23]. In Table IV we display the MPP parameters for the MPP "effective 2-body" potentials MPa$^+$, MPa, MPb, MPc as employed in Ref. [23, 24], and the tuned MPP denoted as MP17 used a standard of the MPP in these notes on the G-matrix calculations. Fig. 5 shows the comparison of MP17 and the MPP parameters used in [23]. We have made equal $V_{MPP}(MP17) = V_{MPP}(MPb)$ for x=1 fm.

TABLE IV: Parameters MP17, MPa$^+$, MPa, MPb, MPc. For MP17 $m_P = 223.12$MeV, and for MPa/b/c $m_P = 234.6$ MeV.

		g_P	g_{3P}	g_{4P}
MP17	:	3.054	3.00	64.0
MPa$^+$	:	3.670	1.31	80.0
MPa	:	3.670	2.34	30.0
MPb	:	3.670	2.94	0.0
MPc	:	3.670	2.34	0.0

X. ΛN,ΣN, AND ΞN G-MATRIX APPLICATION (A)

In this section the couplings for the $(VV)_0$- and $(AA)_0$-pair are for the vector and axial-vector octets V_8 and A_8 respetively.

The ESC08c parameters are used for the coupling constants in these notes are primarily from the fit ESC08c.marius17a with parbbsc.marius17a, denoted as ESC17a. The fitted pair coupling parameters for the NN$\oplus$ YN data are

$$g_{(\pi\pi)_0} = 0,\ g_{\pi\eta} = -0.17249,\ g_{\pi\sigma} = -0.04384,\ g_{\pi\omega} = -0.04779. \tag{10.1}$$

In the three-body "effective two-body" potential we take $g_{(\pi\pi)_0}$, which for clarity we denote by $\hat{g}_{(\pi\pi)_0}$, as an adjustable parameter. (In the future, in order to achieve full consistency, the well-depth's U_Λ, U_Σ, and U_Ξ should be fitted simultaneously with the ESC-model meson and pair couplings.)

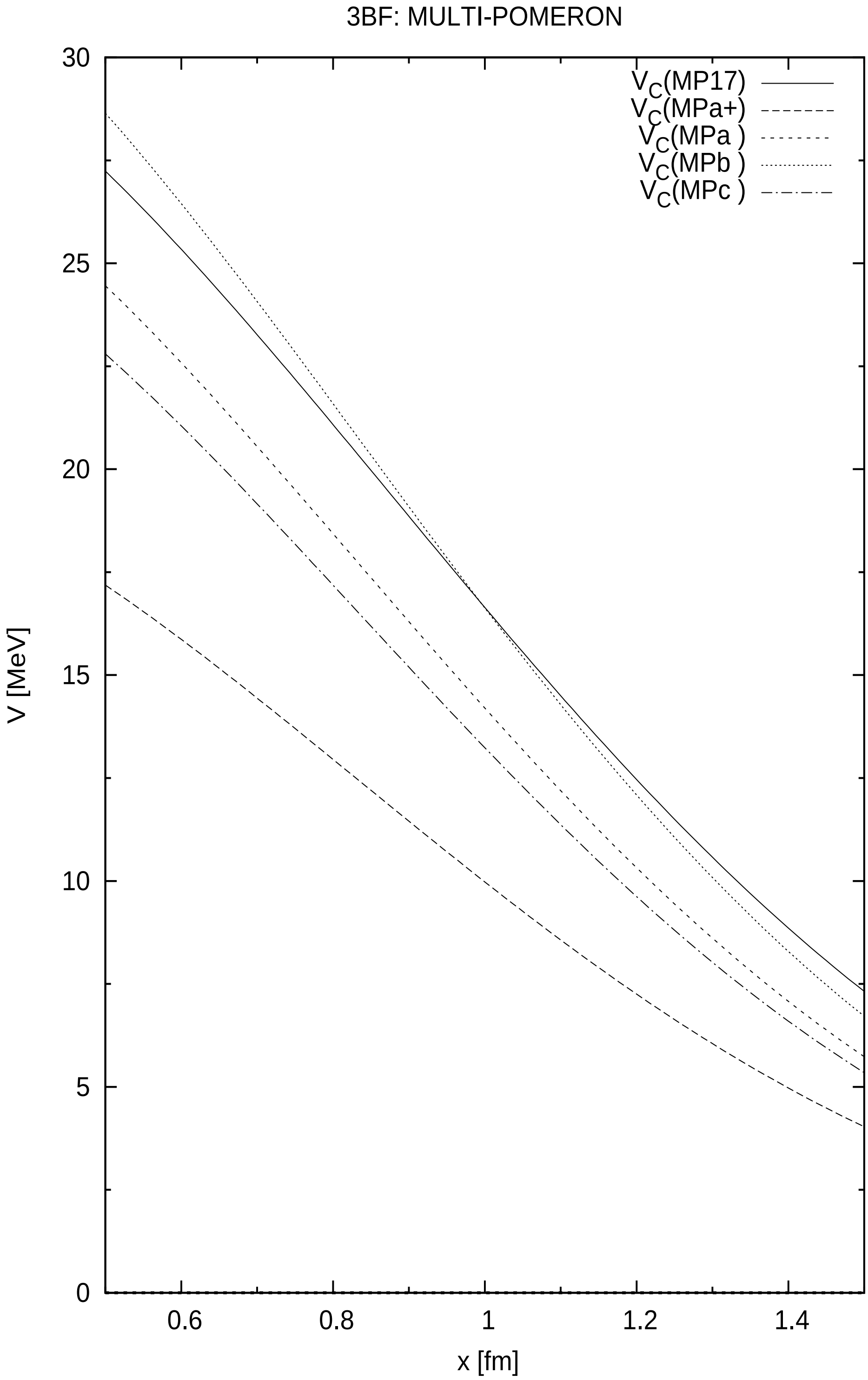


FIG. 5: Comparison MPP Effective 2-body potentials: MP17, MPa^{+}, MPa, MPb, MPc.

In the results in the Tables below are obtained by treating the multi-pomeron triple and quadruple couplings g_{3P} and g_{4P} recpectively as adjustable parameters. Furthermore, we introduced a pair-vertex cut-off parameter Λ_{pr}, which is treated also as adjustable. This makes the following form factor change in the *Effective 2-body potentials*

$$\exp\left[-\frac{\mathbf{k}^2}{\Lambda^2}\right] \Rightarrow \exp\left[-\frac{\mathbf{k}^2}{\Lambda'^2}\right], \quad \Lambda' = \frac{\sqrt{2}\Lambda}{\sqrt{1+2\Lambda^2/\Lambda_{pr}^2}}. \tag{10.2}$$

A. ΛN G-matrix Application

In this section the couplings for the $(VV)_0$- and $(AA)_0$-pair are for the vector and axial-vector octets V_8 and A_8 respetively.

In Table V we display the details for U_Λ for the previous ESC08 model without Three-body *Effective two-body* potentials. In Table VI and Table VII we display the details for U_Λ, U_Σ for

TABLE V: Values of $U_\Lambda(\rho_0)$ and partial wave contributions in $^{2S+1}L_J$ states from the G-matrix calculations (in MeV). The value specified by D gives the sum of $^{2S+1}D_J$ contributions. Contributions from S-state spin-spin interactions are given by $U_{\sigma\sigma} = (U_\Lambda(^3S_1) - 3U_\Lambda(^1S_0))/12$.

	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$	D	U_Λ	$U_{\sigma\sigma}$
ESC08c	−13.3	−25.4	2.6	0.0	1.1	−3.0	−1.6	−39.6	1.22
ESC08c$^+$	−13.3	−25.6	3.0	0.2	1.5	−2.1	−2.3	−38.6	1.19

the GESC017 model, parameters parbbsc.marius17a, to see the effects of the TBF, MPP, and FM-three-body forces. Here, &MPP means ESC17a+MPP etc., and the line TOT means ESC17a+TBF+MPP.

B. ΣN G-matrix Application

In Table VII the partial wave contributions to the Σ-well-depth are shown for a variety of *Effective two-body potentials.*

TABLE VI: GESC17: Values of $U_\Lambda(\rho_0)(K=0)$ and partial wave contributions in ${}^{2S+1}L_J$ states from the G-matrix calculations (in MeV). Parameters Effective 2-body potential: (a) TBF0: $g_{(\pi\pi)_0} = 0, g_{\pi\eta} = -0.17249, g_{\pi\sigma} = -0.04384, g_{\pi\omega} = -0.04779$, and $\Lambda_{pr} = 450$ MeV; (b) MPb: $g_{3P} = 3.0, g_{4P} = 64.0$; (c) FM: $\Lambda_{FM} = 2000.0$ MeV. Extended TBF: (1) TBFa: $g_{(\sigma\sigma)} = 0.15$, (2) TBFb: $g_{(VV)_0} = 0.15$, (3) TBFc: $g_{(AA)_0} = 0.30$. GESC17: $g_{(\sigma\sigma)} = 0.40$, $g_{(VV)_0} = 0.2, g_{(AA)_0} = 0$.

	${}^1S_0$	${}^3S_1$	${}^1P_1$	${}^3P_0$	${}^3P_1$	${}^3P_2$	D	U_Λ	$U_{\sigma\sigma}$
ESC17a	−15.9	−28.0	2.3	−0.1	+1.0	−3.1	−1.3	−45.4	+1.65
&MPb	−6.3	+0.3	4.3	+0.5	+3.4	+1.0	−0.5	+2.5	+1.60
&TBF0	−21.0	−18.1	2.7	−0.6	+2.0	−2.9	−1.6	−39.6	+3.73
&TBFa	−25.3	−34.6	2.0	−1.0	+1.1	−4.6	−1.9	−64.5	+3.44
&TBFb	−20.0	−14.6	2.8	−0.2	+2.1	−1.8	−1.6	−33.4	+3.77
&TBFc	−20.9	−18.0	2.7	−0.6	+2.9	−2.9	−1.6	−39.5	+3.72
TOT	−15.0	+0.5	4.0	+0.3	+3.5	+0.9	−1.0	−7.0	+3.79
GESC17	−21.5	−25.5	2.9	−0.0	+2.0	−1.4	−1.4	−45.2	+3.25
&FM	−15.2	−27.6	2.4	−0.1	+1.1	−3.2	−1.3	−44.2	+1.52

C. ΞN G-matrix Application

In Table VIII the partial wave contributions to the Ξ-well-depth are shown for a variety of *Effective two-body potentials.*

TABLE VII: Values of $U_\Sigma(\rho_0)(K=0)$-partial wave contributions in ${}^{2S+1}L_J$ states from the G-matrix calculations (in MeV). Parameters Effective 2-body potential: (a) TBF0: $g_{(\pi\pi)_0}=0, g_{\pi\eta}=-0.17249, g_{\pi\sigma}=-0.04384, g_{\pi\omega}=-0.04779$, and $\Lambda_{pr}=450$ MeV; (b) MPb: $g_{3P}=3.0, g_{4P}=64.0$, a_{PB}=2.6; (c) FM: $\Lambda_{FM}=2000.0$ MeV. Extended TBF: (1) TBFa: $g_{(\sigma\sigma)}=0.15$, (2) TBFb: $g_{(VV)_0}=0.15$, (3) TBFc: $g_{(AA)_0}=0.30$. GESC17: $g_{(\sigma\sigma)}=0.40$, $g_{(VV)_0}=0.2, g_{(AA)_0}=0$; GSE17a: a_{PB}=1.5, GSE17b: a_{PB}=2.5. n.c.= no convergence 30 steps.

	T	${}^1S_0$	${}^3S_1$	${}^1P_1$	${}^3P_0$	${}^3P_1$	${}^3P_2$	D	U_Σ	Γ_Σ
ESC17a	1/2	+10.5	−25.7	1.9	2.1	−5.4	−0.4	+0.7		
	3/2	−14.3	+38.0	−3.6	−2.2	+5.2	−3.1	−0.2	−7.8	28.5
&MPb	1/2	+9.8	−8.0	2.5	2.3	−3.9	+0.9	−0.4		
	3/2	−5.9	+73.0	−0.1	−1.6	+6.5	+0.1	+0.7	+76.1	n.c.
&TBF0	1/2	+11.4	−18.6	0.7	-3.1	+3.0	−2.1	+0.2		
	3/2	−22.5	+31.5	−2.6	+2.2	−1.6	−0.7	−1.1	−3.5	16.7
&TBFa	1/2	+9.7	−31.9	0.3	-3.5	+2.3	−3.3	+0.0		
	3/2	−29.5	+19.6	−4.1	+1.9	−2.7	−3.2	−0.5	−46.3	19.3
&TBFb	1/2	+11.0	−16.2	0.8	-2.9	+3.1	−1.5	+0.4		
	3/2	−20.3	+33.3	−2.3	+2.6	−1.2	+0.5	−1.1	+5.7	15.5
&TBFc	1/2	+11.4	−18.5	0.7	-3.1	+3.0	−2.1	+0.2		
	3/2	−22.3	+31.4	−2.6	+2.2	−1.6	−0.8	−0.3	−3.3	16.6
TOT	1/2	+13.3	−15.1	1.0	-2.5	+4.3	+0.4	+0.5		
	3/2	−20.3	+48.6	+0.1	+2.6	−0.9	+1.2	−0.7	+32.6	n.c.
GESC17a	1/2	+13.5	−28.1	0.6	-2.6	+4.0	+0.2	+0.3		
	3/2	−29.9	+45.1	−0.4	+2.3	−2.0	−1.2	−0.4	+1.0	20.5
GESC17b	1/2	+13.8	−26.6	0.7	-2.2	+4.8	+1.7	−0.5		
	3/2	−31.9	+56.3	+1.4	+2.3	−2.0	−1.3	−0.2	+16.9	21.6
&FM	1/2	+12.8	−30.6	2.6	2.0	−5.7	−0.9	−0.6		
	3/2	−9.7	+23.8	−2.5	−2.3	+5.0	−3.7	−0.2	−10.1	23.7

TABLE VIII: GESC17: Values of $U_\Xi(\rho_0)$ and partial wave contributions in ${}^{2S+1}L_J$ states from the G-matrix calculations (in MeV). The TBF1a etc. potentials are ESC08 $\oplus$ TBF+MPP. Parameters Effective 2-body potential: (a) TBF0: $g_{(\pi\pi)_0} = 0, g_{\pi\eta} = -0.17249, g_{\pi\sigma} = -0.04384, g_{\pi\omega} = -0.04779$, and $\Lambda_{pr} = 450$ MeV; (b) MPb: $g_{3P} = 3.0, g_{4P} = 64.0$; (c) FM: $\Lambda_{FM} = 2000.0$ MeV. Extended TBF: (1) TBFa: $g_{(\sigma\sigma)} = 0.30$, (2) TBFb: $g_{(VV)_0} = 0.15$, (3) TBFc: $g_{(AA)_0} = 0.30$. TOT$^{*)}$: $g_{(\sigma\sigma)} = 0.6, g_{(VV)} = g_{(AA)} = 0$. TOT$^{**)}$: $g_{(\sigma\sigma)} = 0.6, g_{(VV)} = 0.125, g_{(AA)} = 0$. GESC17= TOT$^{***)}$: $g_{(\sigma\sigma)} = 0.7, g_{(VV)} = 0.2, g_{(AA)} = 0$.

	T	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$	U_Ξ	Γ_Ξ
ESC17a	0	−1.9	−3.2	−0.3	−6.0	1.4	−1.4		
	1	7.9	2.8	1.0	0.6	−2.2	−0.1	−1.3	13.3
&MPb	0	2.0	+6.3	0.6	−2.8	2.2	+0.3		
	1	12.4	27.1	3.5	1.5	+0.5	4.5	+58.1	13.5
&TBF0	0	-8.8	+2.6	-0.0	-1.3	-1.6	−0.2		
	1	9.0	−0.3	+1.0	-0.2	−1.1	-0.6	−1.5	1.7
&TBFa	0	-22.9	−4.7	-0.7	-1.6	-2.3	−1.2		
	1	4.8	-18.4	-0.4	-0.7	−2.7	-3.2	−54.0	0.9
&TBFb	0	-11.1	+3.6	0.1	-1.4	-1.5	−0.1		
	1	10.0	3.4	1.3	-0.1	−0.7	-0.0	+3.5	1.4
&TBFc	0	-8.7	+2.6	-0.0	-1.3	-1.6	−0.2		
	1	9.2	-0.6	1.1	-0.1	−1.3	-0.6	−1.6	1.7
TOT	0	-29.9	−2.1	-0.3	−1.5	-1.9	−0.5		
	1	8.0	-7.5	+1.0	-0.3	−1.3	-0.7	−37.1	0.9$^{*)}$
TOT	0	-28.1	−1.5	-0.1	−1.9	-1.8	−0.4		
	1	8.7	-5.0	+1.2	-0.2	−1.0	-0.3	−30.4	48.3$^{**)}$
GESC17	0	8.6	−3.9	-0.3	−3.8	-1.9	−0.8		
	1	6.4	-11.1	+0.9	-0.4	−1.4	-0.9	−8.6	50.9$^{***)}$

XI. ΛN,ΣN, AND ΞN G-MATRIX APPLICATION (B)

In this section we extend the three-body interactions by the inclusion of the $(\omega_1\omega_1)$*-pair etc. couplings. So we have, next to the pair interactions included in the ESC08-model, also*

$$\begin{aligned}
\mathcal{H}_{(\sigma_1\sigma_1)}(x) &= g_{\sigma_1\sigma_1}\ [\bar{\psi_B}\psi_B]\ \sigma_1^2,\ \ \mathcal{H}_{(\omega_1\omega_1)}(x) = g_{\omega_1\omega_1}\ [\bar{\psi_B}\psi_B]\ \omega_1^2, \\
\mathcal{H}_{(V_8V_8)}(x) &= g_{(V_8V_8)}\ [\bar{\psi_B}\psi_B]\ \left\{\boldsymbol{\rho}\cdot\boldsymbol{\rho} + 2K^{*\dagger}\cdot K^* + \omega_8\omega_8\right\} \\
\mathcal{H}_{(A_8A_8)}(x) &= g_{(A_8A_8)}\ [\bar{\psi_B}\psi_B]\ \left\{\mathbf{A}_1\cdot\mathbf{A}_1 + 2K_A^{*\dagger}\cdot K_A^* + D_{1,8}\ D_{1,8}\right\}.
\end{aligned}$$

Like in the OBE-potentials this σ_1 *and* ω_1 *BB-coupling are not much different, and we can take advantage of the* $\omega-\sigma$ *cancellations.*

In Tables IX we give results for the ESC-parameters parbbsc.marius17a plus those of the extra $(\sigma\sigma), (VV)$ and (AA) pair interactions. In Table X similarly for parbbsc.marius17b. In this last table we also show some results with the inclusion of the Fujita-Miyazawa three-body interactions. The result for ΔU_N, extra attraction ≈ 6 MeV, is in accordance to that given in [9]. Note that the contribution of the FM-potential is attractive to U_Λ, U_Σ, and U_Ξ as well and of the same size as for U_N. These new pair-couplings are very reasonable following the discussion in appendices contained in these notes, taking into account Kleinert's paper and sigma-pomeron cancelations. The U_Σ still is a little enigmatic in the case of parbbsc.marius17a, but not for parbbsc.marius17b!? The ESC08 with parbbsc.marius17a is more attractive than with parbbsc.marius17b.
Note: Always the MPP parameters are fixed G_{3P}=3.0, G_{4P}=64, giving an MPP potential as used for neutron-star matter. The ESC parameters are parbbsc.marius17a (in most cases) and parbbsc.marius17b. The latter fits the NN+YN data best.
Looking at the differences between 17a and 17b it is clear that we can try to find an optimal ESC08c parameter set, which fits the different well-depth's best.
Note that the couplings are rather close together, and in a simultaneous fit with the ESC-parameters possibly can be taken to be the same for the U_N, U_Λ, U_Σ and U_Ξ!? This eventually for the future!

Conclusion: It seems that the inclusion of the three-body effects can give a satisfactory U_Ξ, with dominant attraction in the $\Xi N(^3S_1, T=1)$ partial wave. (In order not to overload this section I do not give here the partial wave contributions, but they look fine to me.) This

TABLE IX: GESC17: Values of E_N, U_Λ, U_Σ and U_Ξ for symmetric nuclear matter with density ρ_0, $k_F = 1.35$ fm. The potentials are ESC08 $\oplus$ TBF+MPP. ESC08 parameters are parbbsc.marius17a. The V_8 and A_8 couplings are taken $g_{(V_8V_8)} = g_{(A_8A_8)} = 2.5$. $E_N = T_N + U_N$, $T_N = 22.676$ MeV. Items with $\star$) means n.c.

$g_{\sigma_1\sigma_1}$	$g_{\omega_1\omega_1}$	E_N	U_Λ	$U_{\sigma\sigma}$	U_Σ	U_Ξ	a_{PB}
0.725	0.70	-15.0	—	—	—	—	
0.735	0.70	-16.2	—	—	—	—	
0.750	0.70	-18.3	—	—	—	—	
0.725	0.70	—	-64.2	2.59	—	—	
0.700	0.70	—	-57.5	2.68	—	—	
0.700	0.80	—	-41.8	2.86	—	—	
0.550	0.80	—-	——	—-	+7.2	—-	0.00
0.550	0.80	—-	——	—-	$+14^{\star)}$	—	0.33
0.550	0.80	—-	——	—-	$+22^{\star)}$	—	1.00
0.600	0.80	—-	——	—-	$+14^{\star)}$	—	2.50
0.700	0.80	—-	——	—-	+2.7	—	2.50
0.700	0.60	——	——	—-	——	+3.7	
0.750	0.60	——	——	—-	——	-9.0	
0.750	0.575	——	——	—-	——	-17.8	
0.750	0.55	——	——	—-	——	-27.2	

with at the same time (i) no deuteron-like b.s. (ruled out by Saclay-Rome-Vanderbilt) and (ii) small ΞN scattering X-sections.

So far we have achieved a model for BB-interactions which consists of two parts: (i) Two-body BB-potentials which describe all BB-scattering data succesfully, and (ii) "Effective two-body" BB-potentials, derived from three-body interactions based on SU(3)-symmetric interactions.

TABLE X: GESC17: Values of U_N, U_Λ, U_Σ and U_Ξ for symmetric nuclear matter with density ρ_0, $k_F = 1.35$ fm. The potentials are ESC08 $\oplus$ TBF+MPP. ESC08 parameters are parbbsc.marius17b. The V_8 and A_8 couplings are taken $g_{(V_8V_8)} = g_{(A_8A_8)} = 0.6$. $E_N = T_N + U_N$, $T_N = 22.676$ MeV. Lines marked by (i) $^{*)}$ are with FM-potential included, (ii) $^{**)}$ FM-potential with D=0, (iii) $^{***)}$ FM-potential A LA [25].

$g_{\sigma_1\sigma_1}$	$g_{\omega_1\omega_1}$	E_N	U_Λ	$U_{\sigma\sigma}$	U_Σ	U_Ξ	Γ_Ξ	a_{PB}
0.000	0.00	-19.7	—	—	—	—		
0.000	0.00	-25.7	—	—	—	—		*)
0.000	0.00	-24.7	—	—	—	—		**)
0.800	0.60	-12.6	—	—	—	—		
0.800	0.55	-16.2	—	—	—	—		
0.800	0.55	-15.3	—	—	—	—		*)
0.800	0.55	-16.1	—	—	—	—		***)
0.000	0.00	—	-47.5	0.61	—	—		
0.000	0.00	—	-53.2	2.10	—	—		*)
0.700	0.70	—	-6.72	1.98	—	—		
0.750	0.60	—	-27.3	1.72	—	—		
0.800	0.60	—	-35.4	1.62	—	—		
0.800	0.55	—	-42.5	1.54	—	—		
0.800	0.55	—	-41.2	0.97	—	—		*)
0.000	0.00	—-	—	—-	-5.5	—-		0.0
0.000	0.00	—-	—	—-	-25.0	—-		0.0 *)
0.600	0.80	—-	—	—-	+65.1	—-		0.0
0.700	0.70	—-	—	—-	+19.6	—		0.0
0.700	0.70	—-	—	—-	+20.9	—		0.0 *)
0.000	0.00	—	—	—-	—	+14.2	11.7	
0.000	0.00	—	—	—-	—	+6.4	5.5	*)
1.200	0.45	—	—	—-	—	-12.5	16.1	
1.500	0.60	—	—	—-	—	-18.7	13.6	
1.500	0.65	—	—	—-	—	-6.48	14.3	
1.200	0.45	—	—	—-	—	-12.5	15.5	*)

TABLE XI: GESC17: Values of U_N, U_Λ, U_Σ and U_Ξ for symmetric nuclear matter with density ρ_0, $k_F = 1.35$ fm. The potentials are ESC08 $\oplus$ TBF+MPP. ESC08 with broad $\kappa(931)$, parameters are parbbsc.kapgam14. The V_8 and A_8 couplings are taken $g_{(V_8V_8)} = g_{(A_8A_8)} = 0.6$. $E_N = T_N + U_N$, $T_N = 22.676$ MeV. Lines marked by: 1) with $g_{V_8V_8)} = 0.90, g_{(A_8A_8)} = 0.60$, 2) with $g_{V_8V_8)} = g_{(A_8A_8)} = 0.0$, and 3) with $g_{V_8V_8)} = 0, g_{(A_8A_8)} = 0.60$. The nucleon energy is $E_T = T_N + U_N, T_N =$ 22.676 MeV.

$g_{\sigma_1\sigma_1}$	$g_{\omega_1\omega_1}$	E_N	U_Λ	$U_{\sigma\sigma}$	U_Σ	U_Ξ	Γ_Ξ	a_{PB}
0.000	0.00	-19.7	—	—	—	—		
0.800	0.55	-50.1	—	—	—	—		
0.600	0.55	-22.6	—	—	—	—		
0.600	0.60	-18.6	—	—	—	—		
0.000	0.00	—	-44.1	1.03	—	—		
0.600	0.60	—	-44.5	2.18	—	—		
0.600	0.60	—	-47.2	2.22	—	—		1)
0.000	0.00	—-	—–	—-	-1.20	—-		0.0
0.500	0.70	—-	—–	—-	+16.0	—-		0.0
0.550	0.75	—-	—–	—-	+8.00	—-		0.0
0.550	0.80	—-	—–	—-	> 10	—-		0.0
0.000	0.00	—–	—–	—-	—–	+19.5	7.7	
0.700	0.25	—–	—–	—-	—–	-17.1	10.3	
0.700	0.30	—–	—–	—-	—–	-3.9	10.3	
0.700	0.30	—–	—–	—-	—–	-12.7	9.85	1)
0.700	0.30	—–	—–	—-	—–	+13.6	11.1	2)
0.700	0.30	—–	—–	—-	—–	+13.5	11.0	3)

XII. NUCLEAR SATURATION APPLICATION

With a change of $g_{(\sigma\sigma)} = 0.4 \to 0.325$, and all other pair-parameters the same as for U_Λ, U_Σ, and U_Ξ, at $k_F = 1.35$ fm^{-1}, for symmetric nuclear matter we obtain $T_N = 22.68$ MeV, $U_N = -38.18$ MeV, giving $E_{TOT} = T_N + U_N = -15.51$ MeV. Experimentally one has

TABLE XII: ΞN low-energy parameters for ESC08c-model with parbbsc.17a-d. The V_8 and A_8 couplings are taken $g_{\sigma_1\sigma_1} = 1.20, g_{\omega_1\omega_1} = 0.45$, and $g_{(V_8V_8)} = g_{(A_8A_8)} = 0.6$.

	$a_{\Lambda\Lambda}(^1S_0)$	$a_{\Xi N}(^1S_0, T=1)$	$a_{\Xi N}(^3S_1, T=1)$	$a_{\Xi N}(^3S_1, T=0)$	U_Ξ	Γ_Ξ
17a :	-1.23	0.44	0.02	-0.40	-101.	0.00
17b :	-0.61	0.52	0.11	-0.83	-12.5	16.1
17c :	-0.59	0.55	0.14	-0.75	-9.56	14.8
17d :	-0.59	0.54	0.11	-0.79	-24.2	28.6

TABLE XIII: GESC17: Values of U_N, U_Λ, U_Σ and U_Ξ for symmetric nuclear matter with density ρ_0, $k_F = 1.35$ fm. The potentials are ESC08 $\oplus$ TBF+MPP. ESC08 with broad $\kappa(931)$, parameters are parbbsc.new17. $\Delta(exp) = U(exp) - U(ESC08 + EFF2)$.

	$\Lambda_{pr} = 450$ MeV	ΔU_N	ΔU_Λ	ΔU_Σ	ΔU_Ξ
Exp.		-37.9	+39.9	+20.0	-15.0
ESC08		-43.9	-44.9	-3.95	+22.5
$\Delta(exp)$		6.0	+5.0	+19.0	-28.0
TBF	$g_{\sigma_1\sigma_1} = 0.3$	-28.5	-50.0	-90.0	-41.0
	$g_{\omega_1\omega_1} = 0.3$	+19.0	+45.0	+56.0	+54.0
	$g_{\omega_8\omega_8} = 0.3$	+0.50	+5.50	+4.50	-2.00
	$g_{a_1a_1} = 0.6$	-1.50	+5.50	+4.80	+2.00
MPP	$g_{3P} = 3.0$	+4.20	+8.40	+6.70	+7.30
	$g_{4P} = 26$	+79.0	+128	+205	+127
FM		-6.00	-7.30	+8.70	+1.70

[26]

$$E_{TOT} = B/A = -16.3 \text{ MeV}, \quad a_{sym} = 32.5 \text{ MeV}.$$

With $g_{(\sigma\sigma)} = 0.40$ we got $E_{TOT} = -21.95$ MeV, and $g_{(\sigma\sigma)} = 0.30$ gives $E_{TOT} = -13.20$ MeV. These variations are due to the sensitivity of U_N, the kinetic contribution T_N remains the same. It is obvious that we can tune the pair-coupling such as to hit the experimental point exactly. The dependence is rather linear, and indeed with $g_{(\sigma\sigma)} = 0.335$ we get for $k_F = 1.35$ fm^{-1}: $T_N = 22.68$ MeV, $U_N = -39.00$ MeV, $E_{TOT} = -16.32$ MeV.

XIII. CONCLUSIONS AND OUTLOOK

The three-body forces presented in these notes lead to "effective" potentials which contains effective pseudoscalar [$(\pi\eta), (\pi\sigma), (\pi\omega)$], scalar [$(\sigma\sigma)$], vector [$(VV)_0$], and axial-vector [$(AA)_0$] type of exchanges, covering the same quantum numbers as in the OBE-models.
In Table XIV the results on the well-depth's in Tables VI, VII, and VIII are assembled to give an overview.

TABLE XIV: Comparison G-matrix well-depths $U_\Lambda, U_\Sigma, U_\Xi$ for symmetric nuclear matter (in MeV).

	U_Λ	$\Gamma_{\sigma\sigma}$	U_Σ	Γ_Σ	U_Ξ	Γ_Ξ
ESC17a	-45.4	$+1.65$	-7.8	28.5	-1.3	13.3
&MPb	$+2.5$	$+1.65$	$+76.1$	n.c.	$+58.1$	13.5
&TBF0	-39.6	$+3.73$	-3.5	16.7	-1.5	1.7.
&TBFa	-64.5	$+3.44$	-46.3	19.3	-54.0	0.9.
&TBFb	-33.4	$+3.77$	$+5.7$	15.5	$+3.5$	1.4
&TBFc	-39.5	$+3.72$	-3.3	16.6	-1.6	1.7.
GESC17	-45.2	$+3.25$	$+16.9$	21.6	-8.6	50.9

Remarks:

(i) The $(\pi\eta)$-pair contribution to $U_{\sigma\sigma}$ is > 0.

(ii) In the ESC08c/ESC16c models is $g_{(\pi\pi)_0} = 0$. So far, we have put $g_{(\pi\pi)_0} = 0$ in ESC-models, but there is no real reason for this.

(iii) From the "effective" two-body potentials, not shown in these notes, one sees that SU(3) symmetry is rather badly broken. This is particularly so for F-M. One would expect $V_{\Sigma\Sigma}(T = 3/2)$ be similar to $V_{NN}(T = 1)$, but that is clearly not the case. The reason is that symmetric nuclear matter is not SU(3) symmetric. In symmetric baryon matter this would not be the case.

(iv) The U_Σ is still unclear. Making a_{PB}=2.0 the iterations did not converge after 30 iterations in some cases. Around ite=16 it seems a bit stable with $U_\Sigma \approx +9$ MeV. The same is true when a_{PB}=2.5, giving $\approx$ 16 MeV.

(v) The Broken-Scale-invariance model [27] and MFT [26] determinations of the $g_{(\sigma\sigma)}, g_{(VV)_0}, g_{(AA)_0}$ couplings are ≈ 1.0. In these derivations the role of the Pomeron was not considered. Taking the $\sigma - P$-cancellations into account one expects the "effective" couplings $\hat{g}_{(\sigma\sigma)}, \hat{g}_{(VV)_0}, \hat{g}_{(AA)_0}$ couplings to be smaller. Therefore, the used couplings in these notes seem quite acceptable.

(v) As for the nuclear saturation the attraction from $(\sigma\sigma)$-pair is proportional to ρ_{NM}, whereas the repulsion from multi-pomeron has besides a ρ_{NM}- also a ρ_{NM}^2-cotribution from the triple- and quartic-vertex respectively. Therefore, at high density like in NS-matter the $(\sigma\sigma)$-pair is (much) weaker than the multi-pomeron.

(vi) In order to give sufficient repulsion for U_Σ the the Pauli-repulsion has to be enhanced, even more that in the ESC08 models.

The well-depth U_Ξ when attractive in the absence of the "new" pair-interactions (notably the $(\sigma\sigma)$-pair), only strong attractive contributions come from the $\Xi N(^3S_1, T=0)$ channel. This is unfavorable for the production of S=-2 hyper-nuclei in a K^-K^+-reaction. The J-PARC E05 experiment is aimed to produce b.s. peaks in the $^{12}C(K^-, K^+)^{12}_{\Xi}$ Be-reaction [28]. The structure and production of p-shell Ξ-hypernuclei has been analyzed by Motoba and Sugimoto [29], see also Motoba et. al. [30]. In this study they with the models NCH-D, Ehime, ESC04d, and ESC08a. As can be seen from the Table VIII models TBF2c and TBF3c are may be similar to ESC04d. Then, the DWIA-spectra in Fig. 7 of Ref. [29] could be expected from application of ESC16c in combination with the TBF and MPP interactions. *With the inclusion of the* $(\sigma\sigma)$*-pair contribution this improves considerably!*

In Table XV gives the comparison of the composition of U_Ξ for the different models. Notice the attraction in $\Xi N(^3S_1, T=1)$-channel for GESC17.

Note: *The total well-depth's get contributions from different partial waves. In hypernuclei and (hyper) nuclear experiments specific partial waves are more important than other ones. For example, in* (K^-, K^+)*-experiments may be selective w.r.t. the* $\Xi N(^3S_1, T=1)$ *wave. Similarly for the twin-hypernuclei of Kanazawa et al.*

Finally: (1) The parameters for GESC17 need further tuning/fitting in order to have one set of parameters for U_Λ, U_Σ, and U_Ξ. The $(\sigma\sigma)$-pair plays an importent rôle in the results obtained in these notes. The $(VV)_0, (AA)_0$ have hardly been explored so far. (2) Consistency ESC and GESC: We introduced an additional cut-off parameter for the pair-vertices. So, in a consistent GESC-model we have to introduce also this pair-vertex cut-off. Moreover, the two-body counterpart of the $(\sigma\sigma), (VV)_0, (AA)_0$ ought to be included.

TABLE XV: Ξ single particle energies U_Ξ and conversion width Γ_Ξ at normal density for ESC04d, ESC08c, and GESC17. Contributions in ${}^{(2T+1)(2S+1)}L_J$ states from the G-matrix calculations (in MeV). The potentials GESC17 = ESC08.17a $\oplus$ TBF+MPP, see Table VIII.

	T	${}^1S_0$	${}^3S_1$	${}^1P_1$	${}^3P_0$	${}^3P_1$	${}^3P_2$	P	U_Ξ	Γ_Ξ
ESC04d	0	6.4	−19.6							
	1	6.4	−5.0					−6.9	−18.7	11.4
ESC08c	0	1.4	−8.0	-0.3	1.8	1.4	−2.1			
	1	10.7	-11.1	1.1	0.7	-2.6	-0.0		−7.0	4.5
GESC17	0	8.6	−3.9	-0.3	-3.8	-1.9	−0.8			
	1	6.4	-11.1	0.9	-0.4	−1.4	-0.9		−8.6	50.9***)

XIV. DISCUSSION

We distinguish three contributions to the *effective two baryon-baryon* potentials: (i) $V(pair)$: pair-contributions; (ii) $V(t_{ch})$: "t-channel" contributions from the two-ps couplings of the $\epsilon(760), S^*(990)$ and pomeron P: and (iii) $V(FM)$: the SU(3)-generalized Fujita-Miyazawa (FM) potentials.
For the meson-pairs that couple to NN, see the interaction Hamiltonians in Appendix G.
In the FM-potentials we use a gaussian form factor with $\Lambda_{FM} = 2\ GeV/c^2$.

The plotted potentials are given for normal nuclear density $\rho_0 = 0.1589\ fm^{-3}$, and without the inclusion of the two-body-correlations [22]. The OBE-couplings are given in Table III. Using these numbers the pure SU(3) octet and singlet couplings are determined in the well known way [31]. The meson-nucleon coupling constants for the relevant (physical) mesons are computed taking into account the meson-mixings, see Appendix G, giving

$$\begin{aligned} Pseudoscalar &: f_{NN\pi} = 0.2686, \quad f_{NN\eta} = 0.1353, \quad f_{NN\eta'} = 0.2691, \\ Vector &: g_{NN\rho} = 0.6280, \quad g_{NN\phi} = -1.2832, \quad g_{NN\omega} = 3.3413, \\ Scalar &: g_{NNa_0} = 0.6129, \quad g_{NNS^*} = -1.5474, \quad g_{NN\epsilon} = 4.0277. \end{aligned} \tag{14.1}$$

The (rationalized) ESC08c NN-pair-couplings are given by

$$\begin{aligned} g_{(\pi\pi)_0} &= 0 \quad , \quad g_{(\pi\eta)_1} = -0.1725, \\ g_{(\pi\sigma)_1} &= -0.0438 \quad , \quad g_{(\pi\omega)_1} = -0.0478, \end{aligned} \tag{14.2}$$

and the coefficients of the pair-interaction Hamiltonians, see Appendix G.

Footnote: The $\sigma - P$ dominance of $g_{(\pi\pi)_0}$ gives

$$g_{(\pi\pi)_0} = \left[\frac{g_{\sigma NN} g_{\sigma\pi\pi}}{m_\sigma^2} - \frac{g_{PNN} g_{P\pi\pi}}{\mathcal{M}^2}\right] f_{NN\pi}^2,$$

where the relative (-)-sign is due the repulsive character of the pomeron P. Assuming chiral-symmetry means that the σ and P contribution nearly cancel.

In Table XVI we give an illustration for the built-up of the χ_{PT} constants c_1 and c_3 using

TABLE XVI: Chiral coefficients c_1 and c_3 from t-channel exchanges and s- and u-channel nucleon and Δ_{33} exchange. For experimental values, see [32].

NLO Chiral-PT Model Pion-nucleon Constants

$c_1(\epsilon)$	=	-14.623 GeV^{-1}	$c_3(\epsilon))$	=	-7.284 GeV^{-1}
$c_1(S^*)$	=	0.588 GeV^{-1}	$c_3(S^*))$	=	0.242 GeV^{-1}
$c_1(P)$	=	19.067 GeV^{-1}	$c_3(P))$	=	3.501 GeV^{-1}
$c_1((\pi\pi)_0)$	=	-0.000 GeV^{-1}	$c_3((\pi\pi)_0))$	=	-0.000 GeV^{-1}
$c_1((\pi\eta)_1)$	=	-2.985 GeV^{-1}	$c_3((\pi\eta)_1))$	=	-0.207 GeV^{-1}
$c_1(ESC08)$	=	+2.047 GeV^{-1}	$c_3(ESC08)$	=	-3.748 GeV^{-1}
$c_1(N_s)$	=	-1.325 GeV^{-1}	$c_3(N_s)$	=	0.000 GeV^{-1}
$c_1(N_u)$	=	-1.325 GeV^{-1}	$c_3(N_u)$	=	0.000 GeV^{-1}
$c_1(\Delta_s)$	=	-1.115 GeV^{-1}	$c_3(\Delta_s)$	=	-0.536 GeV^{-1}
$c_1(\Delta_u)$	=	+0.756 GeV^{-1}	$c_3(\Delta_u)$	=	-0.197 GeV^{-1}
$c_1(tot)$	=	-0.963 GeV^{-1}	$c_3(tot)$	=	-4.482 GeV^{-1}
$c_1(exp)$	=	-0.76 ± 0.07 GeV^{-1}	$c_3(exp)$	=	-5.08 ± 0.28 GeV^{-1}

some recent version of the ESC08-parameters. for the pomeron contributiuons some tuning is done using the $P\pi\pi$-coupling. We note:

(i) The contribution from the pair terms is taken care off in these notes.

(ii) The Δ_{33} s-channel (Δ_s) and u-channel (Δ_u) contributions are supposedly accounted for by the Fujita-Miyazawa (FM) potentials.

(iii) The ϵ, S^* and pomeron P contributions are derived using the Lagrangians $\mathcal{L}(\sigma\pi\pi) = g_{\sigma\pi\pi} m_\pi \ \sigma(\boldsymbol{\pi} \cdot \boldsymbol{\pi})/2$, with $g_\sigma = 10.468, 3.005$, and 3.690 for $\sigma = \epsilon, S^*$ and P respectively. In Fig. 6 the contribution to the effective two-body potential by integrating-out the "third-nucleon" is illustrated. This contribution we denote by $V_{\sigma\pi\pi}$ in the following. $V_{\sigma\pi\pi}$ has the same form as that from the $(\pi\pi)_0$-pair term contribution, see formula (8.1) of the previous section. (NOGTEDOEN)

Note that c_3 is dominated by ESC08c. What about c_1??

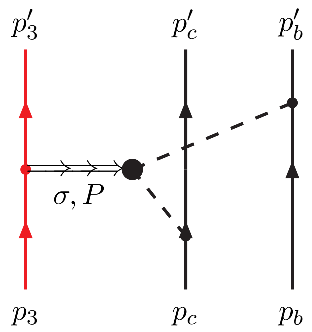
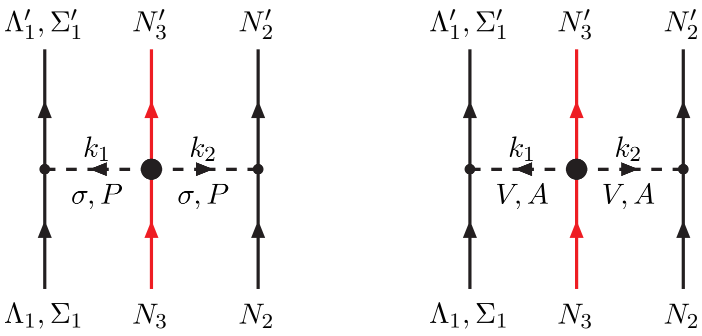


FIG. 6: *Three-body contribution from* $\pi\pi, \eta\eta$, *and* $K\bar{K}$ *couplings of* $\epsilon(760), S^*(990)$ *and P.*

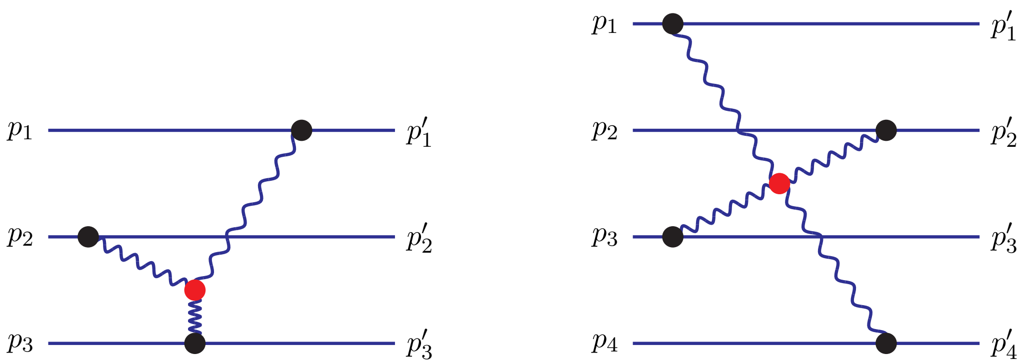
Remark low-energy properties hadron vertices: *The pair-vertices from the* $\sigma \to \sigma\sigma, VV, AA$*-coupling are worked out in Appendices L,J, and K. They lead to "effective" two-body potentials similar to the OBE potentials from the scalar, vector and axial-vector bosons, see text for the scalar and Appendices J and K for the VV and AA respectively. According to the derivations/estimations from broken scale invariance [27] and MFT [26] these couplings are not small. In the Nijmegen OBE and ESC models the sigma-pomeron cancellation is an important factor to keep in line with (broken) chiral-symmetry. Similarly, we ought to consider also the* $P \to \sigma\sigma, VV, AA$*. Therefore, we expect substantial cancellations from these couplings. In this work we do not work out the pomeron related* $\sigma\sigma, VV, AA$ *"effective" potentials. Instead of this we treat the* $(\sigma\sigma)_0-, (VV)_0-, (AA)_0-$*pair couplings as free parameters, which according to the remarks above will be smaller than those derived in [26, 27].*
In line with this, we expect quite large cancellations in diagram (a) between the $\sigma \to \sigma\sigma$ *and similar contributios due to* $P \to PP$*. contributions. Therefore, we use only the* $(\sigma\sigma)$ *part of diagram (a) with an effective coupling* $g_{(\sigma\sigma)_0}$ *which is (quite) smaller than the estimate made in Appendix L. (Note: the PP-contribution in (a) is different in nature from the triple-pomeron contribution.)*

Another example where the $\sigma - P$-cancellation plays possibly an important role is in the triple- and quartic-sigma couplings, see Fig. 8. With positive $g_{3\sigma} > 0, g_{4\sigma} > 0$, the triple scalar-exchange gives an attractive 3BF, which leads eventually to "extraneous" states of nuclear matter [33]. Including also the multi-pomeron vertices, e.g. $\mathcal{L}_3 = g^{(3)}_{\sigma P}[\sigma + P]^3$ large cancellations occur and "extraneous" states can be avoided. (Notice that the quartic $\mathcal{L}_4 = g^{(3)}_{\sigma P}[\sigma + P]^4$ gives repulsion.)



a: $V_{12;3}$ b: $V_{12;3}$

FIG. 7: *Pair-diagrams from $\sigma \to \sigma\sigma, VV, AA$ couplings, and $P \to PP$ coupling (P=pomeron).*



(a) Triple-scalar vertices *(b) Quartic scalar vertices*

FIG. 8: *Multiple sigma and pomeron graphs. The wavy lines stand for either a σ-line or a P-line.*

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APPENDIX A: EXACT REDUCTION DIRAC-SPINORS TO PAULI-SPINORS

The transition from Dirac spinors to Pauli spinors is given here, without approximations. We use the notations $\mathcal{E} = E + M$ and $\mathcal{E}' = E' + M'$, where $E = E(p, M)$ and $E' = E(p', M')$.

Also, we omit, on the right-hand side in the expressions below, the final and initial Pauli spinors $\chi'^\dagger$ and χ respectively, which are self-evident.

$$\bar{u}(\mathbf{p}')u(\mathbf{p}) = +\sqrt{\frac{\mathcal{E}'\mathcal{E}}{4M'M}}\left[\left(1-\frac{\mathbf{p}'\cdot\mathbf{p}}{\mathcal{E}'\mathcal{E}}\right)-i\frac{\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\sigma}}{\mathcal{E}'\mathcal{E}}\right], \tag{A1a}$$

$$\bar{u}(\mathbf{p}')\gamma_5 u(\mathbf{p}) = -\sqrt{\frac{\mathcal{E}'\mathcal{E}}{4M'M}}\left[\frac{\boldsymbol{\sigma}\cdot\mathbf{p}'}{\mathcal{E}'}-\frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{\mathcal{E}}\right], \tag{A1b}$$

$$\bar{u}(\mathbf{p}')\gamma^0 u(\mathbf{p}) = +\sqrt{\frac{\mathcal{E}'\mathcal{E}}{4M'M}}\left[\left(1+\frac{\mathbf{p}'\cdot\mathbf{p}}{\mathcal{E}'\mathcal{E}}\right)+i\frac{\mathbf{p}'\times\mathbf{p}\cdot\boldsymbol{\sigma}}{\mathcal{E}'\mathcal{E}}\right], \tag{A1c}$$

$$\bar{u}(\mathbf{p}')\boldsymbol{\gamma}\, u(\mathbf{p}) = +\sqrt{\frac{\mathcal{E}'\mathcal{E}}{4M'M}}\left[\left(\frac{\mathbf{p}'}{\mathcal{E}'}+\frac{\mathbf{p}}{\mathcal{E}}\right)+i\left(\frac{\boldsymbol{\sigma}\times\mathbf{p}'}{\mathcal{E}'}-\frac{\boldsymbol{\sigma}\times\mathbf{p}}{\mathcal{E}}\right)\right], \tag{A1d}$$

$$\bar{u}(\mathbf{p}')\gamma_5\gamma^0 u(\mathbf{p}) = -\sqrt{\frac{\mathcal{E}'\mathcal{E}}{4M'M}}\left[\frac{\boldsymbol{\sigma}\cdot\mathbf{p}'}{\mathcal{E}'}+\frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{\mathcal{E}}\right], \tag{A1e}$$

$$\begin{aligned}\bar{u}(\mathbf{p}')\gamma_5\boldsymbol{\gamma}\, u(\mathbf{p}) &= -\sqrt{\frac{\mathcal{E}'\mathcal{E}}{4M'M}}\left[\boldsymbol{\sigma}+\frac{(\boldsymbol{\sigma}\cdot\mathbf{p}')\;\boldsymbol{\sigma}\;(\boldsymbol{\sigma}\cdot\mathbf{p})}{\mathcal{E}'\mathcal{E}}\right]\\ &= -\sqrt{\frac{\mathcal{E}'\mathcal{E}}{4M'M}}\left[\left(1-\frac{\mathbf{p}'\cdot\mathbf{p}}{\mathcal{E}'\mathcal{E}}\right)\boldsymbol{\sigma}-i\frac{\mathbf{p}'\times\mathbf{p}}{\mathcal{E}'\mathcal{E}}\right.\\ &\qquad\left.+\frac{1}{\mathcal{E}'\mathcal{E}}\left(\boldsymbol{\sigma}\cdot\mathbf{p}\;\mathbf{p}'+\boldsymbol{\sigma}\cdot\mathbf{p}'\;\mathbf{p}\right)\right]\approx-\boldsymbol{\sigma},,\end{aligned} \tag{A1f}$$

where we defined $\mathbf{k}=\mathbf{p}'-\mathbf{p}$, $\mathbf{q}=(\mathbf{p}'+\mathbf{p})/2$, and $\kappa_V=f_V/g_V$.

Using the the Gordon decomposition

$$i\;\bar{u}(p')\;\sigma^{\mu\nu}(p'-p)_\nu u(p)=\bar{u}(p')\left\{(M'+M)\gamma^\mu-(p'+p)^\mu\right\}\;u(p) \tag{A2}$$

one obtains for the complete vector-vertex

$$\begin{aligned}\bar{u}(p')\Gamma_V^\mu u(p) &\equiv \bar{u}(p')\left[\gamma^\mu+\frac{i}{2\mathcal{M}}\kappa_V\sigma^{\mu\nu}(p'-p)_\nu\right]u(p)\\ &= \bar{u}(p')\left[\left(1+\frac{M'+M}{2\mathcal{M}}\kappa_V\right)\gamma^\mu-\frac{\kappa_V}{2\mathcal{M}}(p'+p)_\mu\right]u(p)\Longrightarrow\end{aligned}$$

$$\begin{aligned}\mu=0\;:\;&+\sqrt{\frac{\mathcal{E}'\mathcal{E}}{4M'M}}\left[\left(1+\frac{M'+M}{2\mathcal{M}}\kappa_V\right)\left(1+\frac{\boldsymbol{\sigma}\cdot\mathbf{p}'\;\boldsymbol{\sigma}\cdot\mathbf{p}}{\mathcal{E}'\mathcal{E}}\right)\right.\\ &\left.-\frac{\kappa_V}{2\mathcal{M}}(E'+E)\left(1-\frac{\boldsymbol{\sigma}\cdot\mathbf{p}'\;\boldsymbol{\sigma}\cdot\mathbf{p}}{\mathcal{E}'\mathcal{E}}\right)\right],\end{aligned} \tag{A3a}$$

$$\begin{aligned}\mu=i\;:\;&+\sqrt{\frac{\mathcal{E}'\mathcal{E}}{4M'M}}\left[\left(1+\frac{M'+M}{2\mathcal{M}}\kappa_V\right)\left\{\left(\frac{\mathbf{p}'}{\mathcal{E}'}+\frac{\mathbf{p}}{\mathcal{E}}\right)+i\left(\frac{\boldsymbol{\sigma}\times\mathbf{p}'}{\mathcal{E}'}-\frac{\boldsymbol{\sigma}\times\mathbf{p}}{\mathcal{E}}\right)\right\}\right.\\ &\left.-\frac{\kappa_V}{2\mathcal{M}}(\mathbf{p}'+\mathbf{p})\left(1-\frac{\boldsymbol{\sigma}\cdot\mathbf{p}'\;\boldsymbol{\sigma}\cdot\mathbf{p}}{\mathcal{E}'\mathcal{E}}\right)\right].\end{aligned} \tag{A3b}$$

APPENDIX B: FOURIER TRANSFORMS NON-LOCAL POTENTIALS II

The Fourier-transform for the operator $(\boldsymbol{\sigma}_1 \cdot \mathbf{k})(\boldsymbol{\sigma}_2 \cdot \mathbf{q})(\mathbf{q} \cdot \mathbf{k})$:

$$V_{ml;nl}(\mathbf{r}', \mathbf{r}) \;=\; \int \frac{d^3q d^3k}{(2\pi)^6}\; k_m k_l q_n q_l\;\; \exp[i\mathbf{q} \cdot (\mathbf{r}' - \mathbf{r})]\;\; \exp[i\mathbf{k} \cdot (\mathbf{r}' + \mathbf{r})/2]\; \widetilde{v}(\mathbf{k}^2). \quad \text{(B1)}$$

The expression to be evaluated is

$$\begin{aligned}(\mathbf{r}'|V_{ml;nl}|\psi) \;&=\; \int d^3r\; (\mathbf{r}'|V_{ml;nl}|\mathbf{r})\psi(\mathbf{r}) = \int d^3a \int \frac{d^3q d^3k}{(2\pi)^6}\; k_m k_l q_n q_l \,\cdot \\ &\quad \times \exp[i\mathbf{q} \cdot \mathbf{a}]\;\; \exp[i\mathbf{k} \cdot (\mathbf{r}' - \mathbf{a}/2)]\; \widetilde{v}(\mathbf{k}^2)\; \psi(\mathbf{r}' - \mathbf{a}) \\ &= \int d^3a\; \left[\int \frac{d^3q}{(2\pi)^3}\; q_n q_l\; e^{i\mathbf{q}\cdot\mathbf{a}}\right]\; w_{ml}(\mathbf{r}' - \mathbf{a}/2)\; \psi(\mathbf{r}' - \mathbf{a}) \\ &= -\int d^3a\, \left[\nabla_{a,n}\nabla_{a,l}\; \delta^3(\mathbf{a})\right] \cdot \left\{w_{ml}(\mathbf{r}' - \mathbf{a}/2)\psi(\mathbf{r}' - \mathbf{a})\right\} \\ &= -\int d^3a\; \delta^3(\mathbf{a})\; \left[\nabla_{a,n}\nabla_{a,l} \left\{w_{ml}(\mathbf{r}' - \mathbf{a}/2)\psi(\mathbf{r}' - \mathbf{a})\right\}\right]. \end{aligned} \quad \text{(B2)}$$

Above, introduced is the variable $\mathbf{a}$

$$\mathbf{a} = \mathbf{r}' - \mathbf{r}, \quad (\mathbf{r}' + \mathbf{r})/2 = \mathbf{r} - \mathbf{a}/2, \quad \text{(B3)}$$

and

$$w_{ml}(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3}\; k_m k_l\; e^{i\mathbf{k}\cdot\mathbf{r}}\; \widetilde{v}(\mathbf{k}^2) = -\nabla_m \nabla_l v(r). \quad \text{(B4)}$$

Working out the final expression in (B2) further is rather standard for the non-local potential terms, see e.g. [34]. One obtains

$$(\mathbf{r}|V_{ml;nl}|\psi) = \frac{1}{4}\left[\nabla_n\nabla_l\; w_{ml}(r)\right]\psi(\mathbf{r}) - \frac{1}{2}\left(\nabla_n\nabla_l w_{ml}(\mathbf{r}) + w_{ml}(\mathbf{r})\nabla_n\nabla_l\right)\; \psi(\mathbf{r}). \quad \text{(B5)}$$

The local part of the potential is

$$V_{local} = -\frac{1}{4}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\nabla})\; (\boldsymbol{\sigma}_2 \cdot \boldsymbol{\nabla})\; \left[\boldsymbol{\nabla}^2 v(r)\right]. \quad \text{(B6)}$$

It is now apparent that the separation between the non-local and local part of the operator of this section, apart from the overall factor $(\boldsymbol{\sigma}_1 \cdot \mathbf{k})$, is as follows:

$$(\mathbf{q} \cdot \mathbf{k})(\boldsymbol{\sigma}_2 \cdot \mathbf{q}) \;=\; \left[(\mathbf{q} \cdot \mathbf{k})(\boldsymbol{\sigma}_2 \cdot \mathbf{q}) + \frac{1}{4}\mathbf{k}^2\; (\boldsymbol{\sigma}_2 \cdot \mathbf{k})\right] - \frac{1}{4}\mathbf{k}^2\; (\boldsymbol{\sigma}_2 \cdot \mathbf{k}). \quad \text{(B7)}$$

APPENDIX C: DIFFERENTIATION FORMULAS

$$(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\nabla})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{\nabla})\; F(r) = \frac{1}{3}\left(\frac{2}{r}F' + F''\right)(r)\; (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + \frac{1}{3}\left(-\frac{1}{r}F' + F''\right)(r)\; S_{12}. \quad \text{(C1)}$$

APPENDIX D: FOURIER INTEGRALS

1. The Fourier transformation for OPE with a Gaussian form factor

$$I_2(m,r) \equiv (2\pi)^{-3} \int d^3k\; e^{i\mathbf{k}\cdot\mathbf{r}} \tilde{I}_2(\mathbf{k}^2)\ , \tag{D1}$$

with

$$\tilde{I}_2(\mathbf{k}^2) = \int_0^\infty d\mu^2 \frac{\rho(\mu^2)}{\mathbf{k}^2+\mu^2} \simeq \frac{e^{-\mathbf{k}^2/\Lambda^2}}{\mathbf{k}^2+m^2}\ , \tag{D2}$$

an approximation discussed in e.g. [19]. The Fourier-transform has been given in [34] with the result

$$\begin{aligned} I_2(m,\Lambda,r) &= \frac{m}{4\pi}\phi_C^0(m,\Lambda,r) \qquad \text{(D3)} \\ \phi_C^0(m,\Lambda,r) &= \exp(m^2/\Lambda^2)\frac{\left[e^{-mr}\ \text{erfc}\ (-\frac{\Lambda r}{2}+\frac{m}{\Lambda}) - e^{mr}\text{erfc}\ (\frac{\Lambda r}{2}+\frac{m}{\Lambda})\right]}{2mr}\ , \end{aligned}$$

where the Erfc-function is defined in [35]. 2. In order to deal with Fourier integrals where $\omega(\mathbf{k})^n, n=1,3,\ldots$ and/or powers of $\omega(\mathbf{k}_1)+\omega(\mathbf{k}_2)$ appear in the denominators, we exploit the following integral-representation [19]

$$\frac{1}{\omega(\mathbf{k})} = \frac{2}{\pi}\int_0^\infty \frac{d\lambda}{\mathbf{k}^2+\mu^2+\lambda^2}\ , \tag{D4}$$

where $\omega(\mathbf{k}) = \sqrt{\mathbf{k}^2+\mu^2}$. Application for $1/\omega(\mathbf{k})$ gives

$$\begin{aligned} \tilde{I}_1(\mathbf{k}^2) &= \int_0^\infty d\mu^2 \frac{\rho(\mu^2)}{\sqrt{\mathbf{k}^2+\mu^2}} = \frac{2}{\pi}\int_0^\infty d\lambda \int_0^\infty d\mu^2 \frac{\rho(\mu^2)}{\mathbf{k}^2+\mu^2+\lambda^2} \\ &\simeq \frac{2}{\pi}\int_0^\infty d\lambda \frac{e^{-(\mathbf{k}^2+\lambda^2)/\Lambda^2}}{\mathbf{k}^2+\mu^2+\lambda^2}\ . \end{aligned} \tag{D5}$$

The Fourier transformation gives

$$I_1(m,r) \equiv (2\pi)^{-3}\int d^3k e^{i\mathbf{k}\cdot\mathbf{r}}\frac{e^{-\mathbf{k}^2/\Lambda^2}}{\omega(\mathbf{k})} = \frac{2}{\pi}\int_0^\infty d\lambda e^{-\lambda^2/\Lambda^2} I_2(\sqrt{m^2+\lambda^2}, r)\ . \tag{D6}$$

Similarly, for the integral where $\omega^3(\mathbf{k})$ occurs in the denominator, using again the integral equation for $1/\omega(\mathbf{k})$, we obtain

$$I_3(m,r) = \frac{2}{\pi}\int_0^\infty \frac{d\lambda}{\lambda^2}\left[I_2(m,r) - e^{-\lambda^2/\Lambda^2} I_2(\sqrt{m^2+\lambda^2}, r)\right]\ . \tag{D7}$$

3. OBE: For integrals with $1/\omega^4(\mathbf{k})$ we find

$$\begin{aligned} \tilde{I}_4(\mathbf{k}^2) &= \int_0^\infty d\mu^2 \frac{\rho(\mu^2)}{(\mathbf{k}^2+\mu^2)^2} = -\frac{d}{d\mathbf{k}^2}\int_0^\infty d\mu^2 \frac{\rho(\mu^2)}{\mathbf{k}^2+\mu^2} \\ &\simeq \frac{e^{-\mathbf{k}^2/\Lambda^2}}{(\mathbf{k}^2+m^2)^2} + \frac{1}{\Lambda^2}\frac{e^{-\mathbf{k}^2/\Lambda^2}}{\mathbf{k}^2+m^2}\ . \end{aligned} \tag{D8}$$

The Fourier transformation gives

$$I_4(m,r) = -\frac{d}{dm^2}\ I_2(m,r) + \frac{1}{\Lambda^2}\ I_2(m,r)\ . \tag{D9}$$

4. TBF: integrals with $1/\omega^4(\mathbf{k})$ we find

$$\tilde{J}_4(\mathbf{k}^2)\ =\ \int_0^\infty d\mu^2 \int_0^\infty d\mu'^{\,2} \frac{\rho(\mu^2)\rho(\mu'^{\,2})}{(\mathbf{k}^2+\mu^2)(\mathbf{k}^2+\mu'^{\,2})} \simeq \frac{e^{-2\mathbf{k}^2/\Lambda^2}}{(\mathbf{k}^2+m^2)^2}, \tag{D10}$$

and the Fourier transformation gives

$$J_4(m,\Lambda/\sqrt{2},r) = -\frac{d}{dm^2} I_2(m,\Lambda/\sqrt{2},r). \tag{D11}$$

5. The derivative w.r.t. the mass:

$$\begin{aligned} J_4(m,\Lambda,r) &= -\frac{1}{2m}\frac{d}{dm} I_2(m,\Lambda,r) = -\frac{m}{4\pi}\left\{\phi^0_C(m,\Lambda,r) - \frac{\Lambda^2}{4m^2}\exp(m^2/\Lambda^2)\cdot\right. \\ &\times \left.\left[e^{-mr}\ \text{erfc}\ (-\frac{\Lambda r}{2}+\frac{m}{\Lambda}) + e^{mr}\text{erfc}\ (\frac{\Lambda r}{2}+\frac{m}{\Lambda})\right]\right\}/\Lambda^2 \equiv -\frac{m}{4\pi}\ \psi^0_C(m,\Lambda,r). \end{aligned} \tag{D12}$$

This follows from the differentiation of the expression (D3):

$$\begin{aligned} \frac{d}{dm}\ I_2 &= \frac{2m}{\Lambda^2}\ I_2 + \frac{m}{4\pi}\left\{-r\left[e^{-mr} Erfc\left(-\frac{\Lambda r}{2}+\frac{m}{\Lambda}\right) + e^{+mr} Erfc\left(+\frac{\Lambda r}{2}+\frac{m}{\Lambda}\right)\right]\right. \\ &-\frac{1}{\Lambda}\left.\left[-e^{-mr}\exp\left[-\left(-\frac{\Lambda r}{2}+\frac{m}{\Lambda}\right)^2\right] + e^{+mr}\exp\left[-\left(+\frac{\Lambda r}{2}+\frac{m}{\Lambda}\right)^2\right]\right]\right\}\cdot e^{m^2/\Lambda^2}/(2mr) \\ &= \frac{2m}{\Lambda^2}\ I_2 - \frac{1}{8\pi}\ e^{m^2/\Lambda^2}\left[e^{-mr} Erfc\left(-\frac{\Lambda r}{2}+\frac{m}{\Lambda}\right) + e^{+mr} Erfc\left(+\frac{\Lambda r}{2}+\frac{m}{\Lambda}\right)\right] \end{aligned}$$

which leads to (D12).

APPENDIX E: TRANSFORMATION TO CONFIGURATION SPACE

The transformation to configuration space for the ESC-potentials is given in Ref. [34]. Here we review this for the $\mathbf{k}^2$ and $(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_1\cdot\mathbf{k})$ terms to establish the formulas used in the subroutines FUNPS2, FUNPHI, and FUNPSI, in the fortran programs. For a local potential we have

$$V(r) = \int \frac{d^3k}{(2\pi)^3}\ e^{i\mathbf{k}\cdot\mathbf{r}}\ \widetilde{v}(\mathbf{k}). \tag{E1}$$

1. **Central potential**: with meson propagator and gaussian form factor

$$\widetilde{V}^{(1)}_C(\mathbf{k}\ =\ \frac{\mathbf{k}^2}{M^2}\ \widetilde{v}^{(1)}(\mathbf{k}) = \int \frac{d^3k}{(2\pi)^3}\frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\mathbf{k}^2+m^2}\frac{\mathbf{k}^2}{M^2}\ e^{-\mathbf{k}^2/\Lambda^2} = -\frac{m}{4\pi}\frac{m^2}{M^2}\ \phi^1_C(m,r,\Lambda), \tag{E2}$$

with

$$\begin{aligned}
\phi^1_C(r) &= \frac{\boldsymbol{\nabla}^2}{m^2}\ \phi^0_C(r) = \frac{1}{m^2}\left(\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}\right)\ \phi^0_C(r) \\
&= \frac{\mu^2}{m^2}\left(\frac{d^2}{dx^2}+\frac{2}{x}\frac{d}{dx}\right)\cdot\frac{\mu}{m}\cdot\widetilde{\phi}^0_C(r)
\end{aligned} \tag{E3}$$

where we introduced μ= pion mass, $x=\mu r$, and $\phi^0_C(r)=(\mu/m)\widetilde{\phi}^0_C(r)$. Then, this central potential is

$$V^{(1)}_C(r) = -\frac{\mu}{4\pi}\ \frac{m^2}{M^2}\cdot\left(\frac{\mu}{m}\right)^2\ \left(\frac{d^2}{dx^2}+\frac{2}{x}\frac{d}{dx}\right)\cdot\widetilde{\phi}^0_C(r) \tag{E4}$$

2. **Tensor potential**: in this case the basic integral is

$$\begin{aligned}
\widetilde{V}^{(0)}_{ij}(\mathbf{k} &= \int\frac{d^3k}{(2\pi)^3}\frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\mathbf{k}^2+m^2}\frac{k_ik_j}{\Lambda^2}\ e^{-\mathbf{k}^2/\Lambda^2} \\
&= -\frac{\nabla_i\nabla_j}{\Lambda^2}\int\frac{d^3k}{(2\pi)^3}\frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\mathbf{k}^2+m^2}e^{-\mathbf{k}^2/\Lambda^2} = -\frac{\nabla_i\nabla_j}{\Lambda^2}\ \frac{m}{4\pi}\ \phi^0_C(r).
\end{aligned} \tag{E5}$$

Now,

$$\begin{aligned}
\nabla_i\nabla_j f(r) &= \left[\left(\nabla_i\nabla_j-\frac{1}{3}\delta_{ij}\boldsymbol{\nabla}^2\right)+\frac{1}{3}\boldsymbol{\nabla}^2\ \delta_{ij}\right]\ f(r) \\
&= \left(\frac{x_ix_j}{r^2}-\frac{1}{3}\delta_{ij}\right)\left(\frac{d^2}{dr^2}-\frac{1}{r}\frac{d}{dr}\right)f(r)+\frac{1}{3}\delta_{ij}\ \left(\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}\right)f(r),
\end{aligned}$$

which leads to the tensor potential

$$\begin{aligned}
V^{(0)}_T(r) &= -\frac{m}{4\pi}\cdot\frac{1}{\Lambda^2}\cdot\left(\frac{d^2}{dr^2}-\frac{1}{r}\frac{d}{dr}\right)\ \phi^0_C(r) \\
&= -\frac{\mu}{4\pi}\cdot\frac{m^2}{\Lambda^2}\cdot\left(\frac{\mu}{m}\right)^2\ \left(\frac{d^2}{dx^2}-\frac{1}{x}\frac{d}{dx}\right)\cdot\widetilde{\phi}^0_C(r)
\end{aligned} \tag{E6}$$

APPENDIX F: LONG RANGE LIMITS OF THE POTENTIALS

APPENDIX G: PAIR COUPLINGS AND $SU(3)$-SYMMETRY

The $SU(3)$ octet and singlet mesons, denoted by the subscript 8 respectively 1, are in terms of the physical ones defined as follows:

(i) Pseudo-scalar-mesons:

$$\begin{aligned}
\eta_1 &= \cos\theta_{PV}\eta'-\sin\theta_{PV}\eta \\
\eta_8 &= \sin\theta_{PV}\eta'+\cos\theta_{PV}\eta
\end{aligned}$$

Here, η' and η are the physical pseudo-scalar mesons $\eta(957)$ respectively $\eta(548)$.

(ii) Vector-mesons:

$$\phi_1 = \cos\theta_V\omega - \sin\theta_V\phi$$
$$\phi_8 = \sin\theta_V\omega + \cos\theta_V\phi$$

Here, ϕ and ω are the physical vector mesons $\phi(1019)$ respectively $\omega(783)$.

Below, $\sigma, \mathbf{a}_0, \mathbf{A}_1, \ldots$ are short-hands for respectively the nucleon densities $\bar{\psi}\psi$, $\bar{\psi}\boldsymbol{\tau}\psi$, $\bar{\psi}\gamma_5\gamma_\mu\boldsymbol{\tau}\psi, \ldots$.

Then, one has the following $SU(3)$-invariant pair-interaction Hamiltonians:

1. $SU(3)$-singlet couplings $S^\alpha_\beta = \delta^\alpha_\beta\sigma/\sqrt{3}$:

$$\mathcal{H}_{S_1PP} = \frac{g_{S_1PP}}{\sqrt{3}}\left\{\boldsymbol{\pi}\cdot\boldsymbol{\pi} + 2K^\dagger K + \eta_8\eta_8\right\}\cdot\sigma \tag{G1}$$

2. $SU(3)$-octet symmetric couplings I, $S^\alpha_\beta = (S_8)^\alpha_\beta \Rightarrow (1/4)Tr\{S[P,P]_+\}$:

$$\begin{aligned}\mathcal{H}_{S_8PP} &= \frac{g_{S_8PP}}{\sqrt{6}}\left\{(\mathbf{a}_0\cdot\boldsymbol{\pi})\eta_8 + \frac{\sqrt{3}}{2}\mathbf{a}_0\cdot(K^\dagger\boldsymbol{\tau}K)\right.\\ &+ \frac{\sqrt{3}}{2}\left\{(K_0^\dagger\boldsymbol{\tau}K)\cdot\boldsymbol{\pi} + h.c.\right\} - \frac{1}{2}\left\{(K_0^\dagger K)\eta_8 + h.c.\right\}\\ &+ \left.\frac{1}{2}f_0\left(\boldsymbol{\pi}\cdot\boldsymbol{\pi} - K^\dagger K - \eta_8\eta_8\right)\right\}\end{aligned} \tag{G2}$$

3. $SU(3)$-octet symmetric couplings II, $S^\alpha_\beta = (B_8)^\alpha_\beta \Rightarrow (1/4)Tr\{B^\mu[V_\mu,P]_+\}$:

$$\begin{aligned}\mathcal{H}_{B_8VP} &= \frac{g_{B_8VP}}{\sqrt{6}}\left\{\frac{1}{2}\left[\left(\mathbf{B}_1^\mu\cdot\boldsymbol{\rho}_\mu\right)\eta_8 + \left(\mathbf{B}_1^\mu\cdot\boldsymbol{\pi}_\mu\right)\phi_8\right]\right.\\ &+ \frac{\sqrt{3}}{4}\left[\mathbf{B}_1\cdot(K^{*\dagger}\boldsymbol{\tau}K) + h.c.\right]\\ &+ \frac{\sqrt{3}}{4}\left[(K_1^\dagger\boldsymbol{\tau}K^*)\cdot\boldsymbol{\pi} + (K_1^\dagger\boldsymbol{\tau}K)\cdot\boldsymbol{\rho} + h.c.\right]\\ &- \frac{1}{4}\left[(K_1^\dagger\cdot K^*)\eta_8 + (K_1^\dagger\cdot K)\phi_8 + h.c.\right]\\ &+ \left.\frac{1}{2}H^0\left[\boldsymbol{\rho}\cdot\boldsymbol{\pi} - \frac{1}{2}\left(K^{*\dagger}\cdot K + h.c.\right) - \phi_8\eta_8\right]\right\}\end{aligned} \tag{G3}$$

4. $SU(3)$-octet a-symmetric couplings I, $A^\alpha_\beta = (V_8)^\alpha_\beta \Rightarrow (-i/\sqrt{2})Tr\{V^\mu[P,\partial_\mu P]_-\}$:

$$\begin{aligned}\mathcal{H}_{V_8PP} &= g_{A_8PP}\left\{\frac{1}{2}\boldsymbol{\rho}_\mu\cdot\boldsymbol{\pi}\times\overleftrightarrow{\partial^\mu}\boldsymbol{\pi} + \frac{i}{2}\boldsymbol{\rho}_\mu\cdot(K^\dagger\boldsymbol{\tau}\overleftrightarrow{\partial^\mu}K)\right.\\ &+ \frac{i}{2}\left(K_\mu^{*\dagger}\boldsymbol{\tau}(K\overleftrightarrow{\partial^\mu}\boldsymbol{\pi}) - h.c.\right) + i\frac{\sqrt{3}}{2}\left(K_\mu^{*\dagger}\cdot\right.\\ &\left.\left.(K\cdot\overleftrightarrow{\partial^\mu}\eta_8) - h.c.\right) + \frac{i}{2}\sqrt{3}\phi_\mu(K^\dagger\overleftrightarrow{\partial^\mu}K)\right\}\end{aligned} \tag{G4}$$

5. $SU(3)$-octet a-symmetric couplings II, $A^\alpha_\beta = (A_8)^\alpha_\beta \Rightarrow (-i/\sqrt{2})Tr\{A^\mu[P,V_\mu]_-\}$:

$$\begin{aligned}\mathcal{H}_{A_8VP} &= g_{A_8VP}\Big\{ \mathbf{A}_1\cdot\boldsymbol{\pi}\times\boldsymbol{\rho} \\ &+ \frac{i}{2}\mathbf{A}_1\cdot\left[(K^\dagger\boldsymbol{\tau}K^*)-(K^{*\dagger}\boldsymbol{\tau}K)\right] \\ &-\frac{i}{2}\left(\left[(K^\dagger\boldsymbol{\tau}K_A)\cdot\boldsymbol{\rho}+(K_A^\dagger\boldsymbol{\tau}K^*)\cdot\boldsymbol{\pi}\right]-h.c.\right) \\ &-i\frac{\sqrt{3}}{2}\left(\left[(K^\dagger\cdot K_A)\phi_8+(K_A^\dagger\cdot K^*)\eta_8\right]-h.c.\right) \\ &+\frac{i}{2}\sqrt{3}f_1\left[K^\dagger\cdot K^*-K^{*\dagger}\cdot K\right]\Big\}\end{aligned} \tag{G5}$$

6. $SU(3)$-singlet a-symmetric couplings II, $A^\alpha_\beta = \delta^\alpha_\beta \Rightarrow (1/\sqrt{3})Tr\{[A^\mu,\partial_\mu P].\sigma$:

$$\mathcal{H}_{A_8VP} = \frac{g_{A_8PS}}{\sqrt{3}}\left\{\mathbf{A}_1\cdot\boldsymbol{\pi}+(K_1^\dagger\cdot K)+(K^\dagger\cdot K_1)+(f_1)_8\ \eta_8)\right\}\cdot\sigma. \tag{G6}$$

The relation with the pair-couplings of [4] and paper I is $g_{S_1PP}/\sqrt{3} = g_{(\pi\pi)_0}/m_\pi$, $g_{A_8VP} = g_{(\pi\rho)_1}/m_\pi$ etc.

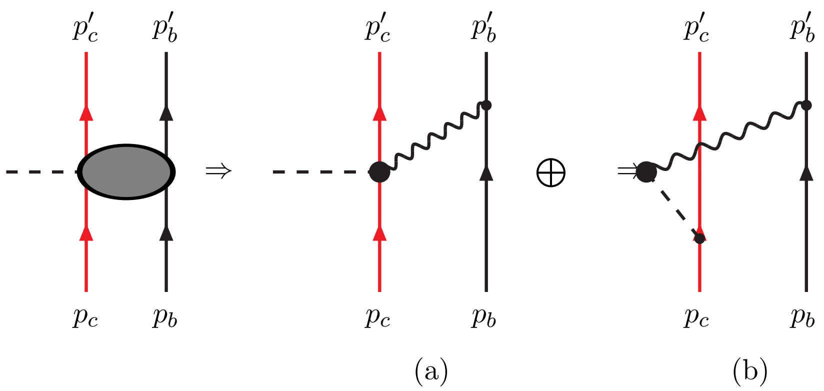

FIG. 9: *Effective Two-body transition potential*

APPENDIX H: SU3-IRREPS AND EFFECTIVE POTENTIALS

In Fig. 9 the connection of the two-body NN, YN, or YY system to the surrounding nuclear medium is scetched. In the contribution (a) the pair-vertex operates on a member of

the BB-system, whereas in (b) the pair-vertex operates on the "third-nucleon". The latter is integrated out. Considering "symmetric" nuclear matter, the dashed line denotes an isospin zero meson. For example η_8, ω_8 or σ. The SU(3) $\{8\}$-meson will cause transition between the SU(3)-irreps of the two-body BB-system. For example transitions $\{27\} \leftrightarrow \{8_s\}$. It will not change the two-partile symmetry of the BB-system. Note that this SU(3)-breaking is there because symmetric nuclear matter in not SU(3)-symmetric. Below, we work out the different cases using the *isoscalar factors* given in [36, 37].
In the first subsection we give the connections between the BB-states and the SU(3)-states. Also, for unbroken SU(3)-symmetry we give the connections for the potentials between the two-baryon- and the SU(3)-irrep-potentials.

1. Effective interactions

In the process of integrating-out and summing over the "third nucleon" in symmetric matter only a few interaction terms in the pair hamiltonians, given in the previous section, contribute effectively. These *effective hamiltonians* are:

$$1.\ \mathcal{H}_{S_1PP} = \frac{g_{S_1PP}}{\sqrt{3}} \left\{\boldsymbol{\pi}\cdot\boldsymbol{\pi} + 2K^\dagger K + \eta_8\eta_8\right\} \cdot \sigma, \tag{H1a}$$

$$2.\ \mathcal{H}_{S_8PP} \Rightarrow \frac{g_{S_8PP}}{\sqrt{6}} \left\{ \frac{1}{2}\left(\boldsymbol{\pi}\cdot\boldsymbol{\pi} - K^\dagger K - \eta_8\eta_8\right) \right\} \cdot f_0, \tag{H1b}$$

$$3.\ \mathcal{H}_{B_8VP} \Rightarrow \frac{g_{B_8VP}}{\sqrt{6}} \left\{ \frac{1}{2}\left(\mathbf{B}_1^\mu \cdot \boldsymbol{\pi}_\mu\right) + \frac{1}{4}(K_1^\dagger \cdot K + K^\dagger \cdot K_1) - \frac{1}{2}H^0\eta_8 \right\} \cdot \phi_8, \tag{H1c}$$

$$4.\ \mathcal{H}_{V_8PP} \Rightarrow g_{A_8PP} \left\{ + \frac{i}{2}\sqrt{3}(K^\dagger \overleftrightarrow{\partial^\mu} K) \right\} \cdot \phi_\mu, \tag{H1d}$$

$$5.\ \mathcal{H}_{A_8VP} \Rightarrow g_{A_8VP} \left\{ - i\frac{\sqrt{3}}{2}\left((K^\dagger \cdot K_A - K_A^\dagger \cdot K\right) \right\} \cdot \phi_8, \tag{H1e}$$

$$6.\ \mathcal{H}_{A_8VP} = \frac{g_{A_8PS}}{\sqrt{3}} \left\{\mathbf{A}_1 \cdot \boldsymbol{\pi} + (K_1^\dagger \cdot K) + (K^\dagger \cdot K_1) + (f_1)_8\ \eta_8)\right\} \cdot \sigma. \tag{H1f}$$

So, apart from the cases 1) and 6), many terms in the pair interaction hamiltonians do not contribute in symmetric nuclear matter. The operator on the "third nucleon" can not contain the isospin or spin operator for a non-zero contribution. Nor can it carry non-zero strangeness. The couplings to the "third nucleon" are mediated only via $\sigma, f_0, \phi_8, \phi_\mu$. These field-operators are written to the right in the formulas (H1). Note that the η_8 coupling gives a spin-operator and hence cannot contribute. Also, we notice that f_0 and ϕ_μ are pair-vertices which couple to the "third nucleon". Furthermore, the H_0-coupling to the "third nucleon" does not contribute because it is pseudoscalar-like and hence brings a spin operator at the vertex. *From the analysis and notes in section VII the interactions terms of 4.*$\mathcal{H}_{V_8PP}$ *and 6.*$\mathcal{H}_{A_8VP}$ *do not contribute.*

APPENDIX I: THE FUJITA-MIYAZAWA-POTENTIAL

The Hamiltonian for the Fujita-Miyazawa pion-nucleon pair-interaction reads [9, 10]

$$\mathcal{H}_{FM} = -\bar{\psi}\left[\left\{\left((A+B)\boldsymbol{\nabla_1}\cdot\boldsymbol{\nabla_2}+D\right)\delta_{ij}-(A-B)\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}_1\times\boldsymbol{\nabla}_2\epsilon_{ijk}\tau_k\right\}\cdot \right.$$
$$\left. \times\pi_{1,i}(x)\pi_{2,j}(x)\right]\psi. \tag{I1}$$

Here the spatial derivatives operate on the pion-fields, and the constants are
Pas eventueel numerieke zaken aan met deze A etc.! oct. 2015

$$A=\frac{5}{18\pi}\int\frac{\sigma_{33}}{\omega_p^2}dp,\quad B=\frac{3}{5}A,\quad D=\frac{2\pi}{3}(a_1+2a_3), \tag{I2}$$

with the numerical values $\int\sigma_{33}/\omega_p^2.dp=3.7m_\pi^{-3}$, and $a_1+2a_3=-0.06m_\pi^{-1}$.

1. The Feynman-diagram Computation FM-potential

Application of the Feynman-rules [20] we have for diagram Fig. 4 and the interaction (I1)

$$-i(2\pi)^4\delta^4(.....)\,\widetilde{V}_{12;3}=(-i)^3\left(\frac{-if_P}{m_\pi}\right)^2\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\cdot$$
$$\times\left[\bar{u}(p_1')\gamma_5(\gamma\cdot p_1'-\gamma\cdot p_1)u(p_1)\right]\cdot\left[\bar{u}(p_2')\gamma_5(\gamma\cdot p_2'-\gamma\cdot p_2)u(p_2)\right]\cdot$$
$$\times\left[\chi_3'\left\{2\left(-(A+B)\mathbf{k}_1\cdot\mathbf{k}_2+D\right)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2+2(A-B)\boldsymbol{\sigma}_3\cdot\mathbf{k}_1\times\mathbf{k}_2\,\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\right\}\chi_3\right]\cdot$$
$$\times\frac{i}{k_1^2-m_\pi^2+i\epsilon}\frac{i}{k_2^2-m_\pi^2+i\epsilon}\cdot(2\pi)^4\delta^4(p_3'-p_3+k_1+k_2)\cdot$$
$$\times(2\pi)^4\delta^4(p_1'-p_1-k_1)(2\pi)^4\delta^4(p_2'-p_2-k_2), \tag{I3}$$

which after performing the k-integrals

$$\begin{aligned}
\widetilde{V}_{12;3} &= -\left(\frac{f_P}{m_\pi}\right)^2 [\bar{u}(p_1')\gamma_5(\gamma\cdot p_1'-\gamma\cdot p_1)u(p_1)]\cdot[\bar{u}(p_2')\gamma_5(\gamma\cdot p_2'-\gamma\cdot p_2)u(p_2)]\cdot \\
&\times\left[\chi_3'\left\{2\left(-(A+B)\mathbf{k}_1\cdot\mathbf{k}_2+D\right)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2+2(A-B)\boldsymbol{\sigma}_3\cdot\mathbf{k}_1\times\mathbf{k}_2\;\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\right\}\chi_3\right]\cdot \\
&\times\frac{1}{(E_{p_1'}-E_{p_1})^2-\omega_1^2+i\epsilon}\frac{1}{(E_{p_2'}-E_{p_2})^2-\omega_2^2+i\epsilon} \\
\Rightarrow &-\left(\frac{f_P}{m_\pi}\right)^2(\boldsymbol{\sigma}_1\cdot\mathbf{k}_1)(\boldsymbol{\sigma}_2\cdot\mathbf{k}_2)\cdot \\
&\times\left[\left\{2\left(-(A+B)\mathbf{k}_1\cdot\mathbf{k}_2+D\right)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2+2(A-B)\boldsymbol{\sigma}_3\cdot\mathbf{k}_1\times\mathbf{k}_2\;\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\right\}\right]\cdot \\
&\times D^0_{pair}(\omega_1,\omega_2). \qquad\qquad \text{(I4)}
\end{aligned}$$

Here, $\omega_{1,2}=\sqrt{\mathbf{k}_{1,2}^2+m_\pi^2}$, and $D^0_{pair}(\omega_1,\omega_2)=1/[\omega_1^2\omega_2^2]$. The last form in (I4) represents the operator for the FM-potential in Pauli-spinor space. Also, due to the $\delta^4(....)$-functions in (I3) one has $\mathbf{k}_1=\mathbf{p}_1'-\mathbf{p}_1$ and $\mathbf{k}_2=\mathbf{p}_2'-\mathbf{p}_2$.

2. FM-pair= $(\pi\pi)_{33}$-Exchange Potential

Here only the $V_{12;3}(\mathbf{k},-\mathbf{k})$-term is non-vanishing. Therefore we obtain, $\mathbf{r}_{12}=\mathbf{x}_1-\mathbf{x}_2$,

$$\begin{aligned}
V^{(eff)}_{FM} &= -2\rho_{NM}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{f_P}{m_\pi}\right)^2\int\frac{d^3k}{(2\pi)^3}e^{-i\mathbf{k}\cdot(\mathbf{x}_1-\mathbf{x}_2)}\cdot \\
&\quad\times(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k})\left[(A+B)\mathbf{k}^2+D\right]\frac{F^2(\mathbf{k}^2)}{(\mathbf{k}^2+m_\pi^2)^2} \\
&= +2\rho_{NM}\left(\frac{f_P}{m_\pi}\right)^2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\nabla}\boldsymbol{\sigma}_2\cdot\boldsymbol{\nabla})\int\frac{d^3k}{(2\pi)^3}e^{-i\mathbf{k}\cdot\mathbf{r}_{12}}\cdot \\
&\quad\times\left[(A+B)\frac{F^2(\mathbf{k}^2)}{(\mathbf{k}^2+m_\pi^2)}+\left(-(A+B)m_\pi^2+D\right)\frac{F^2(\mathbf{k}^2)}{(\mathbf{k}^2+m_\pi^2)^2}\right] \\
&= +2\rho_{NM}\left(\frac{f_P}{m_\pi}\right)^2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\nabla}\boldsymbol{\sigma}_2\cdot\boldsymbol{\nabla})\cdot \\
&\quad\times\left[(A+B)I_2(m_\pi,\Lambda/\sqrt{2},r_{12})+\left(-(A+B)m_\pi^2+D\right)J_4(m_\pi,\Lambda/\sqrt{2},r_{12})\right] \\
&= +2\frac{\rho_{NM}}{m_\pi^3}\left(\frac{f_P^2}{4\pi}\right)\;m_\pi\cdot(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left[(A+B)\;m_\pi^3\left\{\frac{1}{3}\phi^1_C(m_\pi,\Lambda/\sqrt{2},r_{12})(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\right.\right. \\
&\quad\left.+\phi^0_T(m_\pi,\Lambda/\sqrt{2},r_{12})\;S_{12}\right\}+2\left[(A+B)m_\pi^2-D\right]\left(\frac{m_\pi^3}{\Lambda^2}\right)\cdot \\
&\quad\left.\times\left\{\frac{1}{3}\psi^1_C(m_\pi,\Lambda/\sqrt{2},r_{12})\;(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)+\psi^0_T(m_\pi,\Lambda/\sqrt{2},r_{12})\;S_{12}\right\}\right], \qquad \text{(I5)}
\end{aligned}$$

which, using the point limits

$$\lim_{\Lambda\to\infty} I_2(m,r,\Lambda) \ = \ \frac{1}{4\pi}\frac{e^{-mr}}{r}, \quad \lim_{\Lambda\to\infty} I_4(m,r,\Lambda) = \frac{1}{8\pi}\frac{1}{m}e^{-mr}, \tag{I6}$$

gives

$$\begin{aligned}
\lim_{\Lambda\to\infty} V^{(eff)}_{FM} &= +2\rho_{NM}\frac{f_P^2}{4\pi}\left(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right)\left(\boldsymbol{\sigma}_1\cdot\boldsymbol{\nabla}\boldsymbol{\sigma}_2\cdot\boldsymbol{\nabla}\right)\cdot \\
&\quad\times\left[(A+B)\ \frac{e^{-m_\pi r}}{r}+\frac{1}{2m_\pi}\left(-(A+B)m_\pi^2+D\right)e^{-m_\pi r}\right] \\
&= +2\rho_{NM}\frac{f_P^2}{4\pi}\cdot m_\pi^3\cdot\left(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right)\cdot \\
&\quad\times\left[(A+B)\left\{\frac{1}{3}\left(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right)+\left(1+\frac{3}{m_\pi r}+\frac{3}{(m_\pi r)^2}\right)\ S_{12}\right\}\ \frac{e^{-m_\pi r}}{m_\pi r}\right. \\
&\quad+\frac{1}{2}\left(-(A+B)m_\pi^2+D\right)\left\{\frac{1}{3}\left(1-\frac{2}{m_\pi r}\right)\ \left(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right)+\left(1+\frac{1}{m_\pi r}\right)\ S_{12}\right\}\cdot \\
&\quad\left.\times e^{-m_\pi r}\right]
\end{aligned}\tag{I7}$$

3. FM-pair$=(ps-ps)_{33}$-Exchange Potential

Next we consider ps-ps FM-interactions with different masses $m_1\neq m_2$. Again, only the $V_{12;3}(\mathbf{k},-\mathbf{k})$-term is non-vanishing. Similarly as above, we obtain

$$\begin{aligned}
V^{(eff)}_{FM} &= -2\rho_{NM}\left(\frac{f_{P_1}f_{P_2}}{m_\pi^2}\right)\int\frac{d^3k}{(2\pi)^3}e^{-i\mathbf{k}\cdot(\mathbf{x}_1-\mathbf{x}_2)}\left(\boldsymbol{\sigma}_1\cdot\mathbf{k}\right)\left(\boldsymbol{\sigma}_2\cdot\mathbf{k}\right)\cdot \\
&\quad\times\left[(A+B)\ \mathbf{k}^2+D\right]\frac{F^2(\mathbf{k}^2)}{(\mathbf{k}^2+m_1^2)(\mathbf{k}^2+m_2^2)} \\
&= +2\rho_{NM}\left(\frac{f_{P_1}f_{P_2}}{m_\pi^2}\right)\left(\boldsymbol{\sigma}_1\cdot\boldsymbol{\nabla}\boldsymbol{\sigma}_2\cdot\boldsymbol{\nabla}\right)\int\frac{d^3k}{(2\pi)^3}e^{-i\mathbf{k}\cdot\mathbf{r}_{12}}\ \left(m_1^2-m_2^2\right)^{-1}\ F^2(\mathbf{k}^2)\cdot \\
&\quad\times\left\{\frac{\left[(A+B)m_1^2-D\right]}{\mathbf{k}^2+m_1^2}-\frac{\left[(A+B)m_2^2-D\right]}{\mathbf{k}^2+m_2^2}\right\} \\
&= +2\rho_{NM}\left(\frac{f_{P_1}f_{P_2}}{m_\pi^2}\right)\left(\boldsymbol{\sigma}_1\cdot\boldsymbol{\nabla}\boldsymbol{\sigma}_2\cdot\boldsymbol{\nabla}\right)\left(m_1^2-m_2^2\right)^{-1}\cdot \\
&\quad\times\left\{\left[(A+B)m_1^2-D\right]\ I_2(m_1,\Lambda/\sqrt{2},r_{12})-\left[(A+B)m_2^2-D\right]\ I_2(m_2,\Lambda/\sqrt{2},r_{12})\right\} \\
&= +2\frac{\rho_{NM}}{m_\pi^3}\left(\frac{f_{P_1}f_{P_2}}{4\pi}\right)m_\pi\ \left(m_1^2-m_2^2\right)^{-1}\cdot \\
&\quad\times\left[m_1^3\left[(A+B)m_1^2-D\right]\ \left\{\frac{1}{3}\phi^1_C(m_1,\Lambda/\sqrt{2},r_{12})\left(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right)+\phi^0_T(m_1,\Lambda/\sqrt{2},r_{12})\ S_{12}\right\}\right. \\
&\quad\left.-m_2^3\left[(A+B)m_2^2-D\right]\ \left\{\frac{1}{3}\phi^1_C(m_2,\Lambda/\sqrt{2},r_{12})\left(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right)+\phi^0_T(m_2,\Lambda/\sqrt{2},r_{12})\ S_{12}\right\}\right].
\end{aligned}\tag{I8}$$

Here we used the identity

$$\frac{\mathbf{k}^2}{(\mathbf{k}^2+m_1^2)(\mathbf{k}^2+m_2^2)} = \left(m_1^2-m_2^2\right)^{-1}\left\{\frac{m_1^2}{\mathbf{k}^2+m_1^2}-\frac{m_2^2}{\mathbf{k}^2+m_2^2}\right\},$$

so that

$$\frac{(A+B)\mathbf{k}^2+D}{(\mathbf{k}^2+m_1^2)(\mathbf{k}^2+m_2^2)} = \left(m_1^2-m_2^2\right)^{-1}\left\{\frac{(A+B)m_1^2-D}{\mathbf{k}^2+m_1^2}-\frac{(A+B)m_2^2-D}{\mathbf{k}^2+m_2^2}\right\}.$$

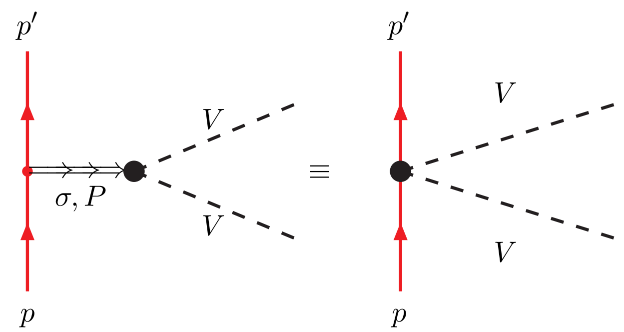

FIG. 10: *Scalar VV-coupling to baryons.*

APPENDIX J: THE σVV-PAIR THREE-BODY POTENTIAL

1. Three-body Matrix Element: The σVV-interaction Hamiltonian density is [27]

$$\mathcal{H}_{\sigma VV}(x) = -g_{\sigma VV}\ m_V\ \sigma(x)V^\mu(x)V_\mu(x). \tag{J1}$$

We note that $g_{\sigma VV}$ has the same sign as in [27]. We define the scalar $(\mathrm{VV})_0$-pair BB-interaction Hamiltonian

$$\mathcal{H}_{(VV)_0}(x) = g_{(VV)_0}\ [\bar{\psi}\psi)(x)]\ V^\mu(x)V_\mu(x)/m_\pi. \tag{J2}$$

Evaluating the diagrams in Fig. 10, including a factor 2 for choice of V of e.g. the incoming vector-meson, we obtain the equation

$$\begin{aligned} -i\langle N',V'|M|N,V\rangle &= -2\frac{(-i)^2}{2!}g_{\sigma NN}g_{\sigma VV}m_V\ [\bar{u}(p')u(p)]\ \frac{i}{k^2-m_\sigma^2+i\epsilon} \\ &= -2i\frac{g_{(VV)_0}}{m_\pi}\ [\bar{u}(p')u(p)] \end{aligned} \tag{J3}$$

which gives, using the low-energy approximation, that

$$g_{(VV)_0} \approx +g_{\sigma NN} g_{\sigma VV} \frac{m_\pi m_V}{2m_\sigma^2}. \tag{J4}$$

Application of the Feynman-rules [20] we have for the diagram (a) in Fig. 2

$$\begin{aligned} -i(2\pi)^4\delta^4(.....)\ V_{12;3} &= (-i)^3\ g_V^2 \frac{g_{(VV)_0}}{m_\pi} \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} [\bar{u}(p_3')u(p_3)] \cdot \\ &\times [\bar{u}(p_1')\ \Gamma_{V,\mu}(p_1',p_1)\ u(p_1)]\ [\bar{u}(p_2')\ \Gamma_V^\rho(p_2',p_2)\ u(p_2)]\ \cdot \\ &\times \frac{-i\eta^{\mu\alpha}}{k_1^2-m_V^2+i\epsilon}\ \frac{-i\eta_{\alpha\rho}}{k_2^2-m_V^2+i\epsilon}(2\pi)^4\delta^4(p_3'-p_3+k_1+k_2)\ \cdot \\ &\times (2\pi)^4\delta^4(p_1'-p_1-k_1)\ (2\pi)^4\delta^4(p_2'-p_2-k_2), \end{aligned} \tag{J5}$$

where

$$\Gamma_V^\mu(p',p) = \gamma^\mu + \frac{i\kappa_V}{2M}\ \sigma^{\mu\nu}(p_1'-p_1)_\nu, \tag{J6}$$

and we also used

$$\sum_\lambda \epsilon_\rho^\alpha(\lambda)\epsilon_\rho^\mu(\lambda) = -\eta^{\mu\alpha} + \frac{k^\mu k^\alpha}{m_\rho^2} \to -\eta^{\mu\alpha}. \tag{J7}$$

and the fact that the vector-meson couples to a conserved current so that the $k^\mu k^\alpha$-term gives no contribution. From (J5) we obtain

$$\begin{aligned} V_{12;3} &= +g_V^2 \frac{g_{(VV)_0}}{m_\pi}\ [\bar{u}(p_3')u(p_3)]\ \cdot \\ &\times [\bar{u}(p_1')\ \Gamma_V^\mu(p_1',p_1)\ u(p_1)]\ [\bar{u}(p_2')\ \Gamma_{V,\mu}(p_2',p_2)\ u(p_2)]\ \cdot \\ &\times \frac{1}{k_1^2-m_V^2+i\epsilon}\frac{1}{k_2^2-m_V^2+i\epsilon}\ , \end{aligned} \tag{J8}$$

where because of the δ-functions in (3.1)

$$p_1' + p_2' + p_3' = p_1 + p_2 + p_3\ , \tag{J9a}$$

$$k_1 = p_1' - p_1\ \ ,\ \ k_2 = p_2' - p_2\ , \tag{J9b}$$

$$p_3' - p_3 = k_1 + k_2\ . \tag{J9c}$$

2. Effective Two-body Potential: Integrating over particle 3 etc. leads to $\mathbf{k}_3 = 0$ and hence $\mathbf{k}_2 = -\mathbf{k}_1$. Neglecting the $1/M$-terms at the vertex of particle 3, i.e. the pair-vertex, leads to

$$\begin{aligned} V_{eff}(12;3) \to\ &+g_V^2\ \frac{g_{(VV)_0}}{m_\pi}\ [\bar{u}(p_1')\ \Gamma_V^\mu(p_1',p_1)\ u(p_1)]\ [\bar{u}(p_2')\ \Gamma_{V,\mu}(p_2',p_2)\ u(p_2)]\ \cdot \\ &\times \left(\frac{1}{k^2-m_V^2+i\epsilon}\right)^2\ \exp\left(-2\mathbf{k}^2/\Lambda^2\right), \end{aligned} \tag{J10}$$

with $k = p_1' - p_1 = p_2 - p_2'$. Note that this is identical in form to OBE vector-exchange, except from the meson denominator. Therefore, the "effective" two-body potential becomes

$$V_{eff}(12;3) = \frac{(4\pi\rho_{NM})}{m_\pi^3}\ \frac{g_{(VV)_0}}{4\pi}\frac{g_V^2}{4\pi}\ m_\pi^2\ \left(-\frac{1}{2m_V}\frac{d}{dm_V}V_V^{(3)}(\mathbf{k}, m_V)\right), \qquad \text{(J11)}$$

where in configuration space, see Ref. [34],

$$\begin{aligned} V_V^{(3)}(r, m_V) \ \approx\ m_V \Bigg\{ & \phi_C^0 + [1 + (\kappa_V + \kappa_V')/2]\frac{m_V^2}{2M'M}\phi_C^1 + (1 + \kappa_V')(1 + \kappa_V)\frac{m_V^2}{6M'M}\phi_C^1(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \\ & -(1 + \kappa_V')(1 + \kappa_V')\frac{m_V^2}{4M'M}\phi_T^0\ S_{12} - [3 + 2(\kappa_V' + \kappa_V)])\frac{m_V^2}{2M'M}\ \phi_{SO}^0\ \mathbf{L} \cdot \mathbf{S} \\ & +[1 + 4(\kappa_V' + \kappa_V)]\frac{m_V^4}{16M'^2M^2}\frac{3}{(m_V r)^2}\ \phi_T^0 Q_{12} \Bigg\} \end{aligned} \qquad \text{(J12)}$$

Here, we neglected the κ_V^2-terms. In the calculations we can used the explicit d/dm_V differentiation or the formula

$$-\frac{1}{2m_V}\frac{d}{dm_V}V_V^{(3)}(\mathbf{k}, m_V) \ = \lim_{m_V' \to m_V}\left[V_V^{(3)}(\mathbf{k}, m_V) - V_V^{(3)}(\mathbf{k}, m_V')\right]/(m_V'^2 - m_V^2).$$

Like in the $V_{(\sigma\sigma)}$-case, also for $V_{(\omega\omega))}$ there is possibly a contribution to the "effective" two-body potential with the pair-coupling in line 1 and 2. Then,

$$V_{(\omega\omega)} \ = \ 2V_\omega(\mathbf{k}, m_\omega)/m_\omega^2 + \lim_{m_\omega' \to m_\omega}\left[V_\omega(\mathbf{k}, m_\omega) - V_\omega(\mathbf{k}, m_\omega')\right]/(m_\omega'^2 - m_\omega^2). \qquad \text{(J13)}$$

This introduces some difference in the treatment of the ω and the other vector mesons! Like in the case of $(\sigma\sigma)$ in the applications we include this potential, of course!

In Ref. [27], Eqn. (11.30), in a study of broken scale-invariance the σVV coupling is derived as being [40]

$$g_{\sigma\pi\pi} = -\gamma\frac{2m_\pi^2}{m_\sigma^2},\ g_{\sigma NN} = -\gamma\frac{M_N}{m_\sigma},\ g_{\sigma\rho\rho} = -\gamma\frac{2m_\rho}{m_\sigma}. \qquad \text{(J14)}$$

The analysis in Ref. [27] comes to the result that instead of $\gamma = -m_\sigma/f_\pi \approx -7$, a better value is $\gamma \approx -5$. This means that $g_{\sigma VV} > 0$. Combining (J4) and (J13) we have

$$\frac{g_{(VV)_0}}{4\pi} = -\frac{\gamma}{\sqrt{4\pi}}\ \frac{g_{\sigma NN}}{\sqrt{4\pi}}\frac{m_\pi m_\rho^2}{m_\sigma^3} \approx +1 > 0! \qquad \text{(J15)}$$

For the SU(3) structure of the σVV-coupling we could choose an SU(3)-singlet coupling (see G1, or an SU(3)-octet symmetric coupling (see G2). Since in Ref. [27] also a $\sigma\rho\rho$-coupling is determined, we choose for the SU(3)-structure the interaction Hamiltonian

$$\mathcal{H}_I = g_{\sigma VV}\left\{\boldsymbol{\rho} \cdot \boldsymbol{\rho} + 2K^{*\dagger} \cdot K^* + \omega_8\omega_8\right\} \cdot \sigma \qquad \text{(J16)}$$

where the $\sigma(x)$-field is an SU(3)-singlet.

Since the pomeron coupling to the ω-meson will be of the same size as that to the σ-meson, we think that the $\sigma-\omega\omega$ coupling for ω_1 will be similar to that to σ_1. Hence, we will introduce two couplings: $g_{(V_1V_1)}$ and $g_{(V_8V_8)}$.

APPENDIX K: THE σAA-PAIR THREE-BODY POTENTIAL

1. Three-body Matrix Element: The σAA-interaction Hamiltonian density is [27]

$$\mathcal{H}_{\sigma AA}(x) = -g_{\sigma AA}\ m_A\ \sigma(x)A^\mu(x)A_\mu(x). \tag{K1}$$

We note that $g_{\sigma AA}$ has the same sign as in [27]. We define the scalar $(\mathrm{AA})_0$-pair BB-interaction Hamiltonian

$$\mathcal{H}_{(AA)_0}(x) = g_{(AA)_0}\ [\bar{\psi}\psi)(x)]\ A^\mu(x)A_\mu(x)/m_\pi. \tag{K2}$$

Again, we evaluate the diagrams in Fig. 10, similar to the σVv-pair in the previous section, which leads to the low-energy approximation

$$g_{(AA)_0} \approx +g_{\sigma NN}g_{\sigma AA}\frac{m_\pi m_A}{2m_\sigma^2}. \tag{K3}$$

The corresponding three-body potential is

$$\begin{aligned} V_{12;3}\ =\ & +g_A^2\frac{g_{(AA)_0}}{m_\pi}\ [\bar{u}(p_3')u(p_3)]\ \cdot \\ & \times [\bar{u}(p_1')\ \Gamma_A^\mu(p_1',p_1)\ u(p_1)]\ [\bar{u}(p_2')\ \Gamma_{A,\mu}(p_2',p_2)\ u(p_2)]\ \cdot \\ & \times \frac{1}{k_1^2-m_A^2+i\epsilon}\frac{1}{k_2^2-m_A^2+i\epsilon}\ , \end{aligned} \tag{K4}$$

with

$$\Gamma_A^\mu(p',p) = \gamma^\mu\gamma_5 + \frac{\kappa_A}{M}\ \gamma_5\ (p'-p)^\mu. \tag{K5}$$

2. Effective Two-body Potential: Integrating over particle 3 etc. leads to $\mathbf{k}_3 = 0$ and hence $\mathbf{k}_2 = -\mathbf{k}_1$. Neglecting the $1/M$-terms at the vertex of particle 3, i.e. the pair-vertex, leads to

$$\begin{aligned} V_{eff}(12;3)\ \rightarrow\ & +g_A^2\ \frac{g_{(AA)_0}}{m_\pi}\ [\bar{u}(p_1')\ \Gamma_A^\mu(p_1',p_1)\ u(p_1)]\ [\bar{u}(p_2')\ \Gamma_{A,\mu}(p_2',p_2)\ u(p_2)]\ \cdot \\ & \times\left(\frac{1}{k^2-m_A^2+i\epsilon}\right)^2\ \exp\left(-2\mathbf{k}^2/\Lambda^2\right), \end{aligned} \tag{K6}$$

with $k = p_1' - p_1 = p_2 - p_2'$. Note that this is identical in form to OBE axial-vector-exchange, except from the meson denominator. Therefore, the "effective" two-body potential becomes

$$V_{eff}(12;3) = \frac{(4\pi\rho_{NM})}{m_\pi^3} \frac{g_{(AA)_0}}{4\pi} \frac{g_A^2}{4\pi} m_\pi^2 \left(-\frac{1}{2m_A}\frac{d}{dm_A} V_A^{(3)}(\mathbf{k}, m_A)\right), \tag{K7}$$

where in configuration space, see Ref. [1, 2],

$$\begin{aligned} V_A^{(3)}(r, m_A) \approx\ & +m_A \left[\left\{\phi_C^0 + [2 + (\kappa_A' + \kappa_A)/2]\frac{m_A^2}{3M'M}\phi_C^1\right.\right. \\ & \left. +\kappa_A'\kappa_A \frac{m_A^4}{12M'^2M^2}\phi_C^2\right\} (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) - \frac{m_A^2}{4M'M}\left\{[1 - 2(\kappa_A' + \kappa_A)]\ \phi_T^0\right. \\ & \left.\left. -\kappa_A'\kappa_A \frac{m_A^2}{M'M}\ \phi_T^1\right\}\ S_{12} + \frac{m_A^2}{2M'M}\ \phi_{SO}^0\ \mathbf{L}\cdot\mathbf{S}\right]. \end{aligned} \tag{K8}$$

In the calculations we can used the explicit d/dm_A differentiation or the formula

$$-\frac{1}{2m_A}\frac{d}{dm_A} V_A^{(3)}(\mathbf{k}, m_A) = \lim_{m_A' \to m_A} \left[V_A^{(3)}(\mathbf{k}, m_A) - V_A^{(3)}(\mathbf{k}, m_A')\right] / (m_A'^2 - m_A^2).$$

In Ref. [27], Eqn. (11.30), in a study of broken scale-invariance the σAA coupling is derived as being [40]

$$g_{\sigma\pi\pi} = -\gamma\frac{2m_\pi^2}{m_\sigma^2},\ g_{\sigma NN} = -\gamma\frac{M_N}{m_\sigma},\ g_{\sigma AA} = -\gamma\frac{2m_A}{m_\sigma}. \tag{K9}$$

The analysis in Ref. [27] comes to the result that instead of $\gamma = -m_\sigma/f_\pi \approx -7$, a better value is $\gamma \approx -5$. This means that $g_{\sigma AA} > 0$. Combining (K4) and (K9) we have

$$\frac{g_{(AA)_0}}{4\pi} = -\frac{\gamma}{\sqrt{4\pi}} \frac{g_{\sigma NN}}{\sqrt{4\pi}} \frac{m_\pi m_A^2}{m_\sigma^3} \approx +2 > 0! \tag{K10}$$

APPENDIX L: THE $\sigma\sigma$-PAIR THREE-BODY POTENTIAL

The σ^3-interaction Hamiltonian density is

$$\mathcal{H}_{3\sigma}(x) = \frac{1}{3!} g_{\sigma\sigma\sigma}\ m_\sigma\ \sigma^3(x). \tag{L1}$$

We define the scalar $(\sigma\sigma)_0$-pair BB-interaction Hamiltonian

$$\mathcal{H}_{(\sigma\sigma)_0}(x) = g_{(\sigma\sigma)_0}\ [\bar{\psi}\psi)(x)]\ \sigma^2(x)/m_\pi. \tag{L2}$$

Evaluating the diagrams in Fig. 10 we obtain the equation [39]

$$\begin{aligned}\langle N',\sigma'|M|N,\sigma\rangle &= g_{\sigma NN}g_{\sigma\sigma\sigma}m_\sigma\ [\bar{u}(p')u(p)]\ \frac{F_{NN\sigma}(k^2)}{k^2-m_\sigma^2+i\epsilon} \\ &= 2\frac{g_{(\sigma\sigma)_0}}{m_\pi}\ [\bar{u}(p')u(p)]\ F_{pr}(k^2),\end{aligned} \tag{L3}$$

which gives, using the low-energy approximation $[\mathbf{k}^2+m_\sigma^2]^{-1}\approx \exp[-\mathbf{k}^2/m_\sigma^2]/m_\sigma^2$,

$$g_{(\sigma\sigma)_0}\approx -g_{\sigma NN}g_{\sigma\sigma\sigma}\frac{m_\pi}{2m_\sigma},\ \ F_{pr}(\mathbf{k}^2)=e^{-\mathbf{k}^2/\Lambda_{pr}^2},\ \ \Lambda_{pr}^2=m_\sigma^2/(1+m_\sigma^2/\Lambda^2). \tag{L4}$$

Comparing with the MFT described in Glendenning [26], p. 147-163 with the interaction Lagrangian $\mathcal{L}_{3\sigma}=-(b/3)\ m_N g^3_{\sigma_1 NN}\ \sigma^3$ [40], gives

$$g_{\sigma\sigma\sigma}=+2m_\sigma^{-1}m_N\ b\ g^3_{\sigma_1 NN},\ \ g_{(\sigma\sigma)_0}=-\frac{m_\pi m_N}{m_\sigma^2}\ b\ g^4_{\sigma NN}. \tag{L5}$$

Here, the SU(3)-singlet σ_1 coupling $g_{\sigma_1 NN}/\sqrt{4\pi}\approx 4.14$ in ESC. Using $m_N=940$ MeV, $m_\pi=140$ MeV, $m_\sigma=760$ MeV, and $b=(0.5-1.5)\times 10^{-2}$ we find $g_{\sigma\sigma\sigma}=(39-120)$ and

$$g_{(\sigma\sigma)_0}/4\pi=-0.23\ (4\pi b)\ [g_{\sigma_1 NN}/\sqrt{4\pi}]^4\approx -(0.34-1.00) \tag{L6}$$

In Ref. [27] formula (8.15), with $f_\pi=95$ MeV

$$g_{\sigma\sigma\sigma}=+3\frac{m_\sigma}{f_\pi}\left(1-\frac{m_\pi^2}{m_\sigma^2}\right)\approx 24. \tag{L7}$$

From Eq. (L4) and (L7) we get

$$g_{(\sigma\sigma)_0}\approx -g_{\sigma NN}\frac{3m_\pi}{2f_\pi}\left(1-\frac{m_\pi^2}{m_\sigma^2}\right)\approx -31.2 \tag{L8}$$

or $g_{(\sigma\sigma)}/4\pi\approx -2.48$.

Analog to the vector and axial-vector case, the "effective" two-body potential becomes

$$V_{eff}(12;3)=\frac{(4\pi\rho_{NM})}{m_\pi^3}\ \frac{g_{(\sigma\sigma)_0}}{4\pi}\frac{g_\sigma^2}{4\pi}\ m_\pi^2\ \left(-\frac{1}{2m_\sigma}\frac{d}{dm_\sigma}V_\sigma^{(3)}(\mathbf{k},m_\sigma)\right), \tag{L9}$$

where in configuration space, see Ref. [34],

$$V_\sigma^{(3)}(r,m_\sigma)\ =\ +m_\sigma\left\{\phi_C^0-\frac{m_\sigma^2}{4M'M}\phi_C^1+\frac{m_\sigma^2}{2M'M}\ \phi_{SO}^0\ \mathbf{L}\cdot\mathbf{S}+\frac{m_\sigma^4}{16M'^2M^2}\frac{3}{(m_\sigma r)^2}\ \phi_T^0 Q_{12}\right\} \tag{L10}$$

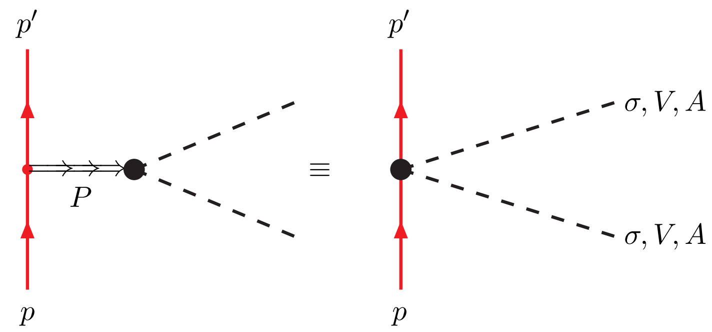

FIG. 11: *Pomeron induced MM-coupling to baryons.*

APPENDIX M: THE POMERON CONTRIBUTION TO $(\sigma\sigma), (VV)_0, (AA)_0$-PAIR COUPLINGS

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The pomeron (P) induced meson-meson (MM) coupling to baryons is illustrated in Fig. 11. Since the color content of all mesons is the same it seems to be justified to assume the *universality* of the PMM-couplings: $g_{P\pi\pi} = g_{P\sigma\sigma} = g_{PVV} = g_{PAA}$.

1. The πN-amplitude from σ- and P-exchange: The $\pi\pi$-coupling of the σ and P are defined by

$$\mathcal{L}_{\sigma\pi\pi} = \frac{1}{2} g_{\sigma\pi\pi} m_\pi [\sigma\ (\boldsymbol{\pi}\cdot\boldsymbol{\pi})],\ \mathcal{L}_{P\pi\pi} = \frac{1}{2} g_{P\pi\pi} m_\pi [P\ (\boldsymbol{\pi}\cdot\boldsymbol{\pi})]. \tag{M1}$$

Then, in terms of the width $g^2_{\sigma\pi\pi}/4\pi = 2(m_\sigma/m_\pi)^2(\Gamma_\sigma/p)$, where $p = \sqrt{m_\sigma^2 - 4m_\pi^2}/2$. In Born-approximation one has

$$\mathcal{M}(t) = m_\pi \left[\frac{g_{\sigma\pi\pi} g_{\sigma NN}}{t - m_\sigma^2} e^{a_\sigma t}(1 + b_\sigma t) + \frac{g_{P\pi\pi} g_{PNN}}{M^2} e^{a_P t} \right], \tag{M2}$$

where a zero in the $NN\sigma$ form factor is included in accordance with the ESC-model. In the following we use the notation

$$G_\sigma^2 =: g_{\sigma\pi\pi} g_{\sigma NN},\ G_P^2 =: g_{P\pi\pi} g_{PNN}. \tag{M3}$$

Since in the "effective" two-body potential $\mathbf{k}_3 = \mathbf{k}_1 + \mathbf{k}_2 = 0$, we take t=0 and the πN amplitude becomes

$$\mathcal{M}^{(0)} = m_\pi \left[-\frac{G_\sigma^2}{m_\sigma^2} + \frac{G_P^2}{M^2} \right]. \tag{M4}$$

The expression of the $\mathcal{M}$ matrix in the $c_{1,3}$ parameters [32, 41] is

$$\mathcal{M}(t) = \frac{1}{M}\left(\frac{m_\pi}{F_\pi}\right)^2 \left\{4(2\tilde{c}_1 - \tilde{c}_3) + 2\tilde{c}_3\frac{t}{m_\pi^2}\right\}, \tag{M5}$$

where $c_i =: \tilde{c}_i/M$, with M=1 GeV/c^2. The results obtained in πN [41] and NN [32] sugggest $\tilde{c}_1 - \tilde{c}_3/2 \approx 1.24$, giving

$$G_P^2\frac{m_\pi^2}{M^2} - G_\sigma^2\frac{m_\pi^2}{m_\sigma^2} \approx 0.8. \tag{M6}$$

2. The Pomeron induced pair couplings: In analogy with the σ-induced pair couplings we have

$$g'_{(\sigma\sigma)} = -g_{PNN}g_{P\sigma\sigma}\frac{m_\pi m_\sigma}{2M^2},\ g'_{(VV)_0} = -g_{PNN}g_{PVV}\frac{m_\pi m_V}{2M^2}, \tag{M7a}$$

$$g'_{(AA)_0} = -g_{PNN}g_{PAA}\frac{m_\pi m_A}{2M^2}, \tag{M7b}$$

where we denoted the P-induced pair NN-couplings as g'. From the universality of the PMM-coupling we have

$$g'_{(VV)_0}/g'_{(\sigma\sigma)} = m_V/m_\sigma,\ g'_{(AA)_0}/g'_{(\sigma\sigma)} = m_A/m_\sigma. \tag{M8}$$

With $g_{PNN}/\sqrt{4\pi} \approx 3.0 \pm 0.5$ and $g_{P\sigma\sigma}/\sqrt{4\pi} \approx 4.3$ we get $g'_{(\sigma\sigma)}/4\pi \approx -0.60 \pm 0.10$. This implies that the total strength of the $(\sigma\sigma)$-pair coupling is $\hat{g}_{(\sigma\sigma)} \approx (1.25 - 0.60) = 0.65$.

APPENDIX N: THE TWO-BODY POTENTIALS FROM $(\sigma\sigma), (VV)_0, (AA)_0$-PAIR COUPLINGS

The two-body pair potentials in the ESC models are restricted to one-pair graphs, and the $(1/M)^2$-terms etc are neglected. The pair-vertex is

$$\mathcal{H}_{MM} = g_{(MM)_0}\left[\bar{\psi}\psi\right]\ M^\mu M_\mu/m_\pi, \tag{N1}$$

where $M^\mu = \sigma, V^\mu$ or A^μ. The two-body potential from the one-pair graph Fig. 12 is

$$\begin{aligned} V^{(1)}_{2,MM} &= g_M^2\frac{g_{(MM)_0}}{m_\pi}\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\ [\bar{u}(p_1')u(p_1)]\cdot \\ &\times[\bar{u}(p_2')\Gamma^\mu_M(p_2', p_2-k_1)\ u(p_2-k_1)]\,[\bar{u}(p_2-k_1)\Gamma_{M,\mu}(p_2-k_1,p_2)\ u(p_2)]\cdot \\ &\times\frac{1}{k_1^2-m_M^2+i\epsilon}\frac{1}{k_2^2-m_M^2+i\epsilon}. \end{aligned} \tag{N2}$$

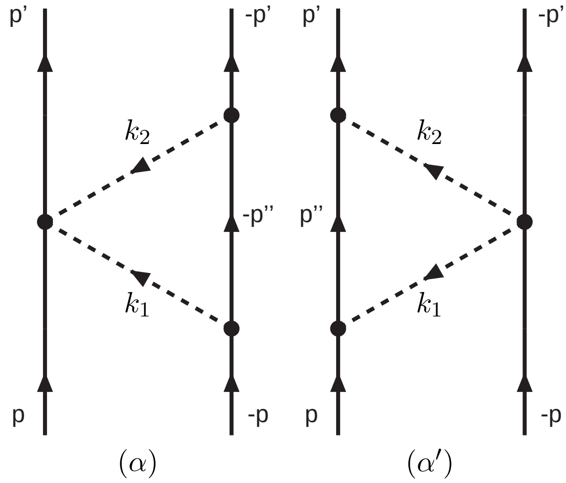

FIG. 12: One-Pair exchange graphs. The 'mirror' graph of (α) is (α').

The reduction of this expression to the matrix elements for the time-ordered graph's in Fig. 13 see [4, 19]. Since for the here considered mesons the matrix elements do not depend on the time ordering the denominators for the diagrams can be added

$$D^{(1)}(\omega_1,\omega_2) \equiv= D_a^{(1)} + D_b^{(1)} + D_c^{(1)} = \frac{1}{\omega_1^2\omega_2^2}. \tag{N3}$$

From Ref. [4] Eq. (3.1) the configuration potentials can be written as

$$\begin{aligned} V^{(1)}_{2,MM}(r) \;=\; & C_{MM} g_M^2 \frac{g_{(MM)_0}}{m_\pi} \int\!\!\int \frac{d^3k_1 d^3k_2}{(2\pi)^6}\; e^{i(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{r}}\;. \\ & \times F_M(\mathbf{k}_1^2) F_M(\mathbf{k}_2^2) O_{MM}(\mathbf{k}_1,\omega_1;\mathbf{k}_2,\omega_2)\; D^{(1)}(\omega_1,\omega_2), \end{aligned} \tag{N4}$$

where C_{MM} contains the isospin and spin-spin factor. Up to first order in (1/M) the scalar pair vertex $\bar{u}u \sim 1$. So, the pair-vertex gives a factor 1 to the O_{MM} operator.

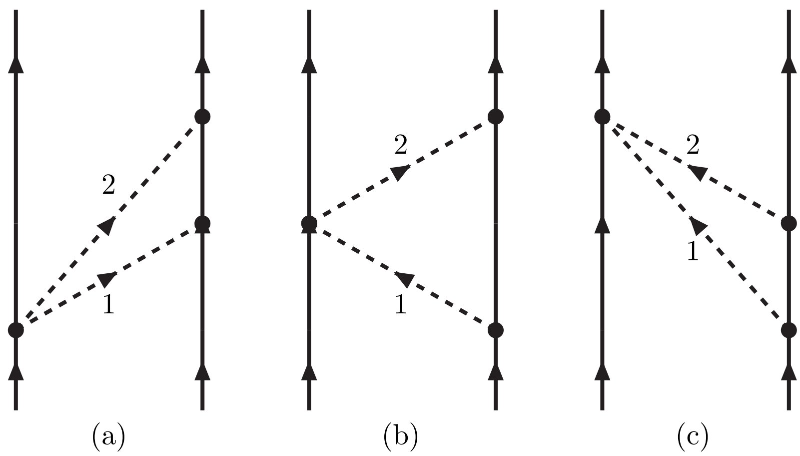

FIG. 13: One-Pair exchange graphs (α). Time-ordered graphs (a,b,c).

1. $V_{2,\sigma\sigma}$: In this case $C_{\sigma\sigma} = 1$ and $\Gamma_S = 1 + (1/M^2) \approx 1$. We get, see Ref. [4], formula (C3),

$$V_{2,\sigma\sigma} = 2 g_{NN\sigma} \frac{g_{(\sigma\sigma)_0}}{m_\pi}\ \left[I_2(m_\sigma, \Lambda_\sigma)\right]^2 . \tag{N5}$$

2. $V_{2,VV}$: From the formulas in (A1) the vector part has a (1/M) at the vector-meson vertex, giving contributions of order $(1/M)^2$. Therefore, only $\mu = 0$ contributes which has $\Gamma_V^0 \sim 1$. So, the contribution is similar to that for $(\sigma\sigma)_0$,

$$V_{2,VV} = 2 C_{VV} g_{NNV} \frac{g_{VV)_0}}{m_\pi}\ \left[I_2(m_V, \Lambda_V, r)\right]^2 . \tag{N6}$$

Here $C_{\rho\rho} = 3$ and $C_{\omega\omega} = 1$.

3. $V_{2,AA}$: From the formulas in (A1) the Γ_A^0 has a (1/M) at the axial-vector-meson vertex, giving contributions of order $(1/M)^2$. Therefore, only $\mu = 0$ contributes which has $\Gamma_A^i \sim -\sigma_i$. Again, this contribution is similar to that for $(\sigma\sigma)_0$,

$$V_{2,AA} = 2 C_{AA} g_{NNA} \frac{g_{AA)_0}}{m_\pi}\ \left[I_2(m_A, \Lambda_A, r)\right]^2 . \tag{N7}$$

Here $C_{A_1A_1} = 9$ and $C_{D_1D_1} = 3$.

(i) These two-body potentials are purely central, i.e. no spin-spin, tensor, etc., and isospin independent. [1] *(ii) The σ, ω etc. contributions have different sign's:*

[1] The one-pair $(1/M^2)$ potentials from the $(\sigma\sigma)$, $(VV)_0$, and $(AA)_0$ graphs are central and spin-orbit. Therefore, for S-waves there are only central-potential contributions.

$g_{(\sigma\sigma)_0} < 0, g_{(VV)_0} > 0, g_{(AA)_0} > 0$. *Therefore, it is possible to have only a small effect on NN scattering but important contributions to the three-body potentials. In particular to the spin-spin and tensor potentials. (iii) Since the pair-vertex is given by SU(3)-singlet σ the st-crossing matrix [38] implies that (ii) extends to all BB-channels.*

To estimate the strength of these two-body potentials we calculate the volume integrals I_V of these potentials. From (N4) one has

$$I_V = \int d^3r\ V_{2,MM}(r) = g_M^2 \frac{g_{(MM)_0}}{m_\pi} \int \frac{d^3k}{(2\pi)^3}\ F_M^2(\mathbf{k}^2)\ D^{(1)}(\omega,\omega) \tag{N8}$$

The integral is

$$\begin{aligned} J_V &= \int \frac{d^3k}{(2\pi)^3}\ F_M^2(\mathbf{k}^2)\ D^{(1)}(\omega,\omega) = \int \frac{d^3k}{(2\pi)^3} e^{-2\mathbf{k}^2/\Lambda^2} \left(\mathbf{k}^2 + m^2\right)^{-2} \\ &= \left(-\frac{d}{dm^2}\right)\left(-\frac{d}{da}\right)\left[\frac{1}{2\pi^2}\int_0^\infty dk\ e^{-ak^2}\left(k^2+m^2\right)^{-1}\right], \end{aligned}$$

where $a = 2/\Lambda^2$. Now

$$J_0 \equiv \left[\cdots\right] = (4\pi m)^{-1}\ e^{am^2}\ Erfc\left(\sqrt{a}m\right) \tag{N9}$$

This gives

$$J(m,\Lambda) = (8\pi m)^{-1}\left(1+\frac{4m^2}{\Lambda^2}\right)\ e^{2m^2/\Lambda^2}\ Erfc\left(\sqrt{2}\frac{m}{\Lambda}\right). \tag{N10}$$

Assuming the (almost) complete cancellation of the $(\sigma\sigma)$ etc volume integrals of the two-body potentials, the ESC08c fit remains the same while there is a considerable "effective" two-body spin-spin, tensor, spin-orbit force. The latter can be most useful to explain the well-depth's.

APPENDIX O: THE σ-DECAY AND EFT LAGRANGIAN

As pointed out by Ko and Rudaz [43] besides the most simple Lagrangian $\mathcal{L}^{(0)}_{\sigma\pi\pi} = g_{\sigma\pi\pi}\sigma\boldsymbol{\pi}\cdot\boldsymbol{\pi}$ also the coupling with two derivatives appears in the linear σ-model Lagrangian, which is useful in keeping the scalar meson width's within reasonable bounds as the scalar mass increases. Also, such couplings and the corresponding contribution to the BB-potentials were considered in the context of an $SU_f(3)$ generalization in [44]. This Lagrangian reads

$\mathcal{L}^{(1)}_{\sigma\pi\pi} = g'_{\sigma\pi\pi}\sigma(\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi})$ Also, such a coupling of the scalar mesons can give an account for the c_3-term in the ($NN2\pi$ effective-field-theory (EFT) interaction Lagrangian [32, 45]

$$\mathcal{L}^{(1)} = -\bar{\psi}\left[8c_1 D^{-1} m_\pi^2 \frac{\boldsymbol{\pi}^2}{F_\pi^2} + 4c_3 \mathbf{D}_\mu\cdot\mathbf{D}^\mu + 2c_4\sigma_{\mu\nu}\boldsymbol{\tau}\cdot\mathbf{D}^\mu\times\mathbf{D}^\nu\right]\psi\ , \tag{O1}$$

where $D = 1+\boldsymbol{\pi}^2/F_\pi^2$ and $\mathbf{D}_\mu = D^{-1}\partial_\mu\boldsymbol{\pi}/F_\pi$, with $F_\pi = 2f_\pi = 185$ MeV. The c_3-term has been determined in e.g. nucleon-nucleon [46]. Notice that because we use the conventions of [20] there is a minus sign in the c_3-term. Since we have found elsewhere that tensor-meson exchange can account only for 20% of the c_3-coefficient, we assume that the remaining part comes from scalar-meson exchange. This is the motivation for the derivation given in this note of the nucleon-nucleon pair-potentials due to the derivative coupling of a scalar $(\pi\pi)_0$-pair coupling.

Application of the c_3-term NN-potential, with a gaussian cut-off $\Lambda \approx 1$ Gev/c^2, to a fit in nucleon-nucleon, using the ESC-model, reveals that it is impossible to reach a sensible description of the NN-phases when we fix the value at $c_3 = -5$ GeV^{-1}, obtained in [46]. This is caused by the large oscillations of this potential below 1 fm. Only by making Λ much smaller it should be possible to use such a potential in the ESC-model. In view of this fact, we analyze an interpretation of this interaction in terms of scalar and diffractive exchange, using an expansion of the πN-amplitude valid for low t-values. It turns out that the c_3-term can be ascribed largely to a form factor effect in the NN-system. As such this interaction is to a large extend effectively already contained implicitly in the ESC-model.
The relation between the pair-coupling parameters $g^{(1,2)}_{(\pi\pi)_0}$ and the EFT-coefficients $c_{1,3}$ is

$$g^{(1)}_{(\pi\pi)_0} = 8\big(m_\pi/F_\pi\big)^2 c_1,\ g^{(2)}_{(\pi\pi)_0} = -4\big(m_\pi/F_\pi\big)^2 c_3, \tag{O2}$$

where $F_\pi = 185$ MeV. With $c_1 = -0.76\pm0.07$ and $c_3 = -4.70\pm1.16$ the couplings would be $g^{(1)}_{(\pi\pi)_0} = -0.43$ and $g^{(2)}_{(\pi\pi)_0} = +1.70$. As remarked already above, since effects of this couplings are implicitly included in the ESC-models it is to be seen how strong these couplings in ESC-models turn out to be.

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[1] Th.A. Rijken, Phys. Rev. **C73**, 44007 (2006).

[2] Th.A. Rijken and Y. Yamamoto, Phys. Rev. **C73**, 44008 2006.

[3] Th.A. Rijken and V.G.J. Stoks, Phys. Rev. **54** (1996) 2851.

[4] Th.A. Rijken and V.G.J. Stoks, Phys. Rev. **54** (1996) 2869.

[5] Th. A. Rijken and J.-W. Wagenaar, *Notes on Relativistic πN-amplitudes with Dynamical Pair Suppression*, THEF-NIJM 09.04, http://nn-online.org/eprints.

[6] Th. A. Rijken and J.-W. Wagenaar, *Notes on Relativistic πN-amplitudes with Dynamical Pair Suppression*, THEF-NIJM 09.04, http://nn-online.org/eprints.

[7] H. Primakoff and T. Holstein, Phys. Rev. **55**, 1218 (1939).

[8] S.D. Drell and Kerson Huang, Phys. Rev. **91**, 1527 (1953).

[9] J.-I. Fujita and H. Miyazawa, Progr. Theor. Phys. **17**, 360 (1957).

[10] H. Miyazawa, Phys. Rev. **104**, 1741 (1956).

[11] B.H.J. McKellar and R. Rajamaran, Phys. Rev. **C3**, 1877 (1971).

[12] S.A. Coon, B.R. Barrett, M.D. Scadron, D.W.E. Blatt, and B.H.J. McKellar, in *Few-Body dynamics*, edited by A.N. Mitra *et al* (North-Holland, Amsterdam, 1976), p.739; S.A. Coon, M.D. Scadron, P.C. McNamee, B.R. Barrett, D.W.E. Blatt, and B.H.J. McKellar, Nucl. Phys. **A317**, 242 (1979).

[13] P. Grangè, A. Lejeune, M. Martzolff, and J.-F. Mathiot, Phys. Rev. **C40** 1040 (1989).

[14] M. Lacombe, B. Loiseau, J.M. Richard, Vinh Mau, J.C. Coté, and R. de Tourreil, Phys. Rev. **C 21**, 861 (1980).

[15] V.G.J. Stoks, R.A.M. Klomp, M.C.M. Rentmeester, and J.J. de Swart, Phys. Rev. **C 48** (1993) 792; R.A.M. Klomp, private communication.

[16] Th.A. Rijken, M.M. Nagels, and Y. Yamamoto, Progr. Theor. Phys. Suppl. **185** (2010) 14.

[17] D. Harrington, Phys. Rev. **141**, 1494 (1966).

[18] J.D. Chalk, III, and B.W. Downs, Phys. Rev. **132**, 2727 (1963)

[19] Th.A. Rijken, Ann. Phys. (N.Y.) **208** (1991), 253-298; Revised version, /http://nn-online.org/eprints/, THEF-NIJM 90.07, january 2002.

[20] We follow the conventions of J.D. Bjorken and S.D. Drell, *Relativistic Quantum Mechanics* and *Relativistic Quantum Fields* (McGraw-Hill Inc., New York; 1965).

[21] B.A. Loiseau, Y. Nogami, and C.K. Ross, Nucl. Phys. **A165** (1971) 601.

[22] T. Ueda, T. Sawada, and S. Takagi, Nucl. Phys. **A285** (1977) 429.

[23] Y. Yamamoto, T.Furumoto, N.Yasutake and Th.A. Rijken, Phys. Rev. **C 88** (2013),

022801(R).

[24] Y. Yamamoto, T.Furumoto, N.Yasutake and Th.A. Rijken, Phys. Rev. **C 90** (2014), 045805.

[25] T. Kasahara, Y. Akaishi, and H. Tanaka, Supp;. Progr. Thoer. Phys. **56**, 96 (1974).

[26] N.K. Glendenning, *Compact Stars, Nuclear Physics, Particle Physics, and General Relativity*, Astronomy and Astrophysics Library, Springer 1997.

[27] H. Kleinert, Fortschritte d. Physik, **21**, 1-55 (1973).

[28] T. Nagae, Progr. Theor. Phys. Suppl. No 185, 299 (2010).

[29] T. Motoba and S. Sugimoto, Nucl. Phys. **A 835**, 223 (2010)

[30] T. Motoba *et al*, Nucl. Phys. **A 827**, 453c (2009).

[31] M. Gell-Mann and Y. Ne-emann, "The Eightfold Way", Benjamin Pub., New York, 1964.

[32] M.C.M. Rentmeester, *Applied Nucleon-Nucleon Partial Wave Analyses*, PhD-thesis, Kath. Univer. Nijmegen, 2001.

[33] T.D. Lee, Rev. Mod. Phys. **47** (1975) 267-275.

[34] M.M. Nagels, T.A. Rijken, and J.J. de Swart, Phys. Rev. D**17**, 768 (1978).

[35] M. Abramowitz and I.A. Stegun, editors, *Handbook of Mathematical Functions* (Dover Publications Inc., New York; 1970).

[36] J.J. de Swart, Rev. Mod. Phys. **35**, 916 (1963); *ibid* **37**, 326 (1965).

[37] P.A. Carruthers, *Introduction to Unitary Symmetry*, Interscience Publishers 1966. See Table A-I, p.192-193.

[38] J.J. de Swart, Nuovo Cimento **31** (1965) 420. *ibid* **37**, 326 (1965).

[39] Here, a factor 2 comes from $H_I^2 = (H_{NN\sigma} + H_{\sigma^3})^2 \to 2H_{NN\sigma}H_{\sigma^3}$.

[40] We define the couplings in the $\mathcal{H}_I$, whereas in [27] and [26] the couplings are defined in $\mathcal{L}_I$ which means a relative (-)-sign.

[41] P. Büttiker and U.-G. Meissner, Nucl. Phys. **668**, 97 (2000)

[42] The Jacobian-coordinates with a proper normalization are given by $P = (p_a + p_b + p_c)/\sqrt{6}$, $p = (p_a - p_b)/2$, $q = (p_a + p_b - 2p_c)/2\sqrt{3}$. Now in the overall CM-system $\mathbf{P} = 0$, and we have the three-dimensional vectors for the particles which can expressed in the two independent vectors $(\mathbf{p}, \mathbf{q})$: $\mathbf{p}_a = \mathbf{p} + \mathbf{q}/\sqrt{3}$, $\mathbf{p}_b = -\mathbf{p} + \mathbf{q}/\sqrt{3}$, and $\mathbf{p}_c = -2\mathbf{q}/\sqrt{3}$. In the $(\mathbf{p}, \mathbf{q})$-plane, taking as unit vectors $\mathbf{e}_1 = \mathbf{p}, \mathbf{e}_2 = \mathbf{q}$ the end-points of the vectors $\mathbf{p}_a, \mathbf{p}_b, \mathbf{p}_c$ are the corners of a regular triangle. The symmetries of the regular triangle are: three refections and three rotations, which constitute the six group elements of D(3). In this way, the symmetry-group

of this triangle is isomorphic to $D(3) \propto S(3)$, and provides a two-dimensional representation $\underline{\mathbf{2}}$ of the permutation group S(3) of the three-particle momenta. The two-dimensional matrices providing the representation are easily obtained.

[43] P. Ko and S. Rudaz, Phys. Rev. **D 50** (1994) 6877.

[44] V.G.J. Stoks and Th.A. Rijken, Nucl. Phys. A **613** (1996) 311.

[45] C. Ordóñez and U. van Kolck, Phys. Lett. B **291** (1992) 459; C. Ordóñez, L. Ray, and U. van Kolck, Phys. Rev. Lett. **72** (1994) 1982, Phys. Rev. C **53** (1996) 2086.

[46] M.C.M. Rentmeester, R.G.E. Timmermans, J.L. Friar, and J.J. de Swart, Phys. Rev. Lett. **82** (1999) 4992. .