

## Electromagnetic Corrections to the One-Pion-Exchange Potential\*

U. van Kolck<sup>1</sup>, M. C. M. Rentmeester<sup>2,5</sup>, J. L. Friar<sup>3,4</sup>☺, T. Goldman<sup>3</sup>, and J. J. de Swart<sup>5</sup>

<sup>1</sup> *Department of Physics, University of Washington, Seattle, WA 98195-1560*

<sup>2</sup> *Department of Physics, The Flinders University of South Australia, Bedford Park, SA 5042, Australia*

<sup>3</sup> *Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545*

<sup>4</sup> *Institute for Nuclear Theory, University of Washington, Seattle, WA 98195-1550*

<sup>5</sup> *Institute for Theoretical Physics, University of Nijmegen, Nijmegen, The Netherlands*

### Abstract

Leading-order electromagnetic loop corrections to the one-pion-exchange potential are computed within the framework of chiral perturbation theory. These corrections generate an effective nucleon-nucleon potential,  $V_{\pi\gamma}$ , which supplements the sum of OPEP and the nucleon-nucleon Coulomb potential. This potential is charge dependent and its construction is demonstrated to be gauge invariant. The potential  $V_{\pi\gamma}$  has been included in the Nijmegen partial-wave analysis of  $np$  data. A particular renormalization scheme is chosen that leads to a negligible change in the  $\pi^\pm NN$  coupling constant and in the  $np$   $^1S_0$  scattering length and effective range.

PACS numbers: 13.75.Cs, 12.38.Bx, 13.40.Ks, 13.75.Gx

Typeset using REVTeX

---

\*Published in Phys. Rev. Lett. **80**, 4386–4389 (1998)

Isospin violation is an august topic[1, 2] in nuclear physics, and considerable progress has been made in understanding the consequences of isospin violation in nuclei, particularly in few-nucleon systems[3, 4]. Nevertheless, a comparable understanding of the underlying mechanisms for isospin violation in the nuclear force has been slower in developing. It is our purpose here to present for the first time complete analytic results for one such mechanism obtained using chiral perturbation theory, and to determine its effect on the  $\pi^\pm NN$  coupling constant and the  $np$   $^1S_0$  scattering length and effective range.

It has been known for decades that the various mechanisms responsible for isospin violation could be graded in strength, with charge-dependent (CD) nuclear forces being generally stronger than charge-symmetry-breaking (CSB) nuclear forces, while strongest of all is the long-range Coulomb force between protons. Only recently, however, have these empirical observations been linked quantitatively to microscopic strong-interaction mechanisms based on symmetries[5, 6, 7].

The strength of isospin violation can be understood[5] using dimensional power counting, a property associated with chiral effective Lagrangians[6]. This technique expresses the strength of any nuclear interaction in terms of several energy scales: a heavy scale  $M_{\text{QCD}} \sim 1$  GeV (the characteristic QCD mass) and the lighter scales  $f_\pi = 92.4$  MeV (the pion decay constant),  $m_\pi = 139.6$  MeV (the scale of explicit chiral-symmetry breaking), and  $Q$  (the effective momentum in a nucleus, which can be taken to be  $\sim m_\pi$ ). In addition, up-down quark-mass-difference-induced isospin violation[7] carries an extra factor of  $\epsilon = (m_d - m_u)/(m_d + m_u) \sim 0.3$ , while electromagnetic interactions carry powers of the fine-structure constant  $\alpha$  ( $\sim 1/137$ ). Dimensionless strong-interaction constants of “natural”[8] size  $\sim 1$  (such as  $g_A = 1.26$ , the axial-vector coupling constant) are ignored. The general technique is not dependent on any particular model, but specific force models can be analyzed and expressed in terms of these quantities, as well. Signs are not determined by such arguments.

Using these techniques one finds that the dominant (isospin-conserving) component of the nuclear force in light nuclei, OPEP, has a strength[9] (corresponding to an energy shift)  $\sim 15$  MeV/nucleon pair, while the  $pp$  Coulomb interaction (CD and CSB) is roughly 1 MeV in strength. The small Breit corrections to the latter (viz., magnetic and velocity-dependent forces) have a nominal 20 keV strength, as does the effect of the nucleon-mass difference. The difference in pion masses in OPEP generates a CD nuclear force of roughly 1/2 MeV in strength. Simultaneous exchange of a pion and a photon between nucleons has a nominal 30 keV strength, as does the electromagnetic modification (at the quark level)[10] of the  $\pi NN$  coupling constant; both are CD mechanisms. The leading-order CSB force generated by the effect of differing quark masses on the  $\pi NN$  coupling constants may be somewhat larger  $\sim 90$  keV. These estimates (which could easily vary by a factor of two) correspond rather well to those obtained from the well-studied isospin violation in few-nucleon systems[3, 4, 10]. Thus, one finds that CD is larger than CSB, and both of these mechanisms are larger than the tiny isospin-violating forces of Type IV [1], all of which was first demonstrated in Ref. [5] using dimensional power counting.

Isospin violation in the nuclear force has also been studied as a natural extension of the partial-wave analysis (PWA) of both  $pp$  and  $np$  scattering data[11], together with the tiny amount of  $nn$  scattering information[12]. It has long been known that the  $NN$   $^1S_0$  scattering lengths are all different, and their differences scale roughly as the interaction strengths

estimated above. These PWAs have allowed the  $\pi^0 pp$ ,  $\pi^0 np$ , and  $\pi^\pm NN$  coupling constants to be separately determined[13], and the Goldberger-Treiman (GT) discrepancy[14] to be obtained, as well. At the present time[13, 10] both the CD and CSB  $\pi NN$  coupling constants are consistent with zero (within  $\lesssim 1\%$  overall uncertainty in the  $\pi NN$  constants), and the GT discrepancy ( $d - 1$ ), which is isospin conserving but chiral-symmetry breaking, is approximately 2%.

The Nijmegen PWA[11] incorporates pion-mass differences in OPEP ( $V_\pi$ ), nucleon-mass differences, and the full electromagnetic interaction[15] (including vacuum polarization) when it determines the  $\pi NN$  coupling constants. Only the effective  $\pi$ - $\gamma$ -exchange interaction has been missing, and we have seen that its strength is comparable to that of several isospin-violating mechanisms normally incorporated. The nominal strength of that force is (as we shall see)  $(\alpha/\pi)V_\pi$ , and this is comparable to the uncertainty in the  $\pi^\pm NN$  coupling constant,  $f_c^2 = (g_A m_{\pi^+} d/2f_\pi)^2/4\pi$ . Therefore, the task that remains is to calculate the effective  $\pi$ - $\gamma$ -exchange force, incorporate it into the Nijmegen PWA, and redetermine  $f_c^2$ .

The calculation is performed within the framework of chiral perturbation theory[5, 6, 16] to leading order in  $1/M_{\text{QCD}}$ . This allows us to make the static approximation for nucleons ( $M_N \rightarrow \infty$ ), and restricts us to the leading-order terms (in a chiral expansion) in the effective Lagrangian. Because we are generating EM corrections to one-pion exchange between nucleons (short-range forces were discussed previously[5, 10]), only single-loop corrections involving photons are required. In static order, the necessary elements of the leading-order interaction Lagrangian are:

$$L^{(0)} = -\frac{g_A}{f_\pi} \bar{N} [\boldsymbol{\sigma} \cdot \nabla (\mathbf{t} \cdot \boldsymbol{\pi})] N + \frac{eg_A}{f_\pi} \bar{N} \boldsymbol{\sigma} \cdot \mathbf{A} (\mathbf{t} \times \boldsymbol{\pi})_3 N - eA_0 \bar{N} (\frac{1}{2} + t_3) N - eA^\mu (\boldsymbol{\pi} \times \partial_\mu \boldsymbol{\pi})_3 + \frac{1}{2} e^2 A^2 (\boldsymbol{\pi}^2 - \pi_3^2) + \dots, \quad (1)$$

where we have used Cartesian notation for the pion charge states ( $\pi_i$ ) but have otherwise conformed to the notation and conventions of Refs. [17] and [10]. Additional terms in the Lagrangian that are not required here can be found in the latter reference. The nucleon (Pauli) spin operator is  $\boldsymbol{\sigma}$ , its isospin operator is  $t_\alpha$  ( $t_3 = \pm \frac{1}{2}$ ),  $e$  is the fundamental (proton) charge, and  $A^\mu$  is the photon field. Although to leading (static) order the  $\pi\gamma N$  seagull interaction involves only  $\mathbf{A}$  and the nucleon EM interaction involves only  $A_0$ , the full pion EM interaction is required.

The calculation requires all diagrams of Fig. (1). Figure (1a) subsumes 4 separate diagrams (a bubble on each nucleon leg), (1b) 2 diagrams, (1c) 4 diagrams, (1d) 2 diagrams, (1e) and (1f) each 2 diagrams, (1g) is unique, while (1h) is accompanied by an EM tadpole (obtained from the last term in Eq. (1)) that is not shown. Altogether, 19 diagrams were separately calculated in both Coulomb and Feynman gauges. All graphs differ in the two gauges. Coulomb gauge has the advantage of eliminating *a priori* any infrared divergences, but considerable complexity is introduced into the calculation. Infrared divergences arise in Feynman gauge but cancel, while a complicated set of terms that ultimately cancel in Coulomb gauge never arise in Feynman gauge. The sum of diagrams is the same in both gauges, a natural consequence of any complete calculation, rather than one focusing on a few selected mechanisms.

In order to obtain physical and useful results we have to use a renormalization prescription. We follow Ref. [18] and keep only nonanalytic terms in divergent loops, incorporating

all analytic terms into the definitions of the coupling constants and masses. The hard parts of vertex loops renormalize the CD  $\pi NN$  coupling constant (denoted by  $\bar{\beta}_{10}$  in Refs. [5, 10]), for example. In addition, diagrams (1e) and (1f) contain a component corresponding to the iteration of the static OPEP and Coulomb potential (in the general case) and this has been subtracted from the full amplitude, since it is automatically included when solving the Schrödinger equation. The remaining amplitude ( $S_{\pi\gamma}$ ) is still gauge invariant and the relation

$$S_{\pi\gamma} = -i V_{\pi\gamma} \quad (2)$$

defines the effective  $\pi$ - $\gamma$ -exchange potential,  $V_{\pi\gamma}$ , between two nucleons arbitrarily labelled 1 and 2. Its isospin structure allows only charged-pion exchange and therefore affects only  $np$  scattering. The usual OPEP corresponding to charged-pion ( $\pi_c$ ) exchange,  $V_{\pi_c}$ , is similarly obtained. Their sum is given by the simple momentum-space expressions

$$V_{\pi\gamma}(\mathbf{q}) + V_{\pi_c}(\mathbf{q}) = -\frac{g_A^2}{f_\pi^2 m_\pi^2} (\mathbf{t}_1 \cdot \mathbf{t}_2 - t_1^3 t_2^3) (\boldsymbol{\sigma}_1 \cdot \mathbf{q} \boldsymbol{\sigma}_2 \cdot \mathbf{q}) [V_{\pi\gamma}(\beta) + V_{\pi_c}(\beta)], \quad (3)$$

$$V_{\pi\gamma}(\beta) = \frac{\alpha}{\pi} \left[ -\frac{(1-\beta^2)^2}{2\beta^4(1+\beta^2)} \ln(1+\beta^2) + \frac{1}{2\beta^2} - \frac{2\bar{\gamma}}{1+\beta^2} \right], \quad (4)$$

where OPEP is determined from

$$V_{\pi_c}(\beta) = \frac{1}{1+\beta^2}, \quad (5)$$

and  $\beta = q/m_\pi$ ,  $\mathbf{q}$  is the momentum transferred between the nucleons,  $m_\pi$  is the charged-pion mass, and  $\bar{\gamma}$  will be discussed below. Details of this calculation (including graph-by-graph results), pion and nucleon  $\sigma$ -terms, and the renormalization-scale dependence of the coupling constants will be published elsewhere.

This force is CD and remarkably simple in form, given the number of processes that contribute. Fourier transformation to configuration space yields

$$V_{\pi\gamma}(\mathbf{r}) = \frac{g_A^2 m_\pi^3}{4\pi f_\pi^2} \left[ \frac{\alpha}{\pi} \right] (\mathbf{t}_1 \cdot \mathbf{t}_2 - t_1^3 t_2^3) \boldsymbol{\sigma}_1 \cdot \nabla_z \boldsymbol{\sigma}_2 \cdot \nabla_z \left[ \frac{I(z)}{z} \right], \quad (6)$$

$$I(z) = 2e^{-z} [\ln(z/2) + \gamma_E - \bar{\gamma}] + 2e^z Ei(-2z) - Ei(-z) \left( 3 + \frac{z^2}{2} \right) + \frac{e^{-z}}{2} (1-z), \quad (7)$$

where  $Ei(-z) = -\int_z^\infty dt e^{-t}/t$  is the exponential integral,  $z = m_\pi cr/\hbar$ ,  $\gamma_E$  is Euler's constant (0.577...), and the usual OPEP corresponding to charged-pion exchange is obtained by substituting  $e^{-z}$  for  $I(z)$  and dropping the factor of  $\alpha/\pi$  in brackets. Our renormalization prescription defines  $\bar{\gamma}$  and is discussed next. The first term in  $I(z)$  (in brackets) determines the asymptotic form, while the volume integral of  $I(z)$  is proportional to  $(\frac{7}{4} - 2\bar{\gamma})$ . A positive value of  $\bar{\gamma}$  ( $\sim 1$ ) weakens both.

The usual definition[13] of the  $\pi NN$  coupling constant requires an extrapolation to the pion pole (i.e.,  $q^2 = m_\pi^2$ ) in the unphysical region of the NN scattering amplitude, with the residue defining the coupling constant and the difference between  $q^2 = 0$  and  $q^2 = m_\pi^2$  defining the GT discrepancy[14]. This pole corresponds (since  $q^0 = 0$  in the static limit)

to  $\beta^2 = -1$ . Although  $V_{\pi_c}(\beta)$  has a simple pole at  $\beta^2 = -1$ , the first term in  $V_{\pi\gamma}(\beta)$  does not (the residue diverges) because of the logarithm induced by the infrared structure of the photon loops. This is reflected in the configuration space term:  $e^{-z} \ln(z)$ . Any constant multiple of  $e^{-z}$  in  $I(z)$  (i.e., a multiple of OPEP) can be arbitrarily transferred to OPEP with an appropriately redefined  $f_c^2$ , since the sum of  $V_{\pi\gamma}$  and  $V_{\pi_c}$  remains unchanged. Alternatively, the logarithmic (asymptotic) term in  $V_{\pi\gamma}(\beta)$  can be made to vanish at any convenient point. Straightforward development of  $V_{\pi\gamma}(\beta)$  leads to  $\bar{\gamma} = 0$  (an even simpler result!). We choose, however, to remove the  $\gamma_E$  term in  $V_{\pi\gamma}(\mathbf{r})$  by performing a further finite renormalization and fixing  $\bar{\gamma} \equiv \gamma_E$ ; this **defines**  $f_c^2$  in the presence of EM corrections, and is analogous to the  $\overline{MS}$  (Modified Minimal Subtraction) renormalization commonly used in Standard Model calculations. The bracketed (logarithmic) term in  $I(z)$  now vanishes at  $r = 2\hbar/m_\pi c = 2.8$  fm. As we have previously discussed, this weakens both the tail of the  $\pi$ - $\gamma$  potential and the volume integral of  $I(z)$  (by a factor of three).

It is difficult to compare our result with previous calculations[19, 20, 21]. None of the CD calculations[22, 23, 24, 25, 26, 27] were complete, and few were written in an easily interpretable form. Some numerical results are available, however[28]. The gauge invariance of our final result gives us confidence in its form. Figure (1g) is easily calculated in old-fashioned perturbation theory in the static limit (see Ref. [28]), and a comparison checks our overall sign and factors. As an additional check, the Breit interaction[29] can be calculated using our conventions and yields the usual result.

The tail of  $V_{\pi\gamma}$  (for  $r > 1.4$  fm) was incorporated into the Nijmegen PWA of only the  $np$  data[30] on an equal footing with OPEP. A total of 4107  $np$  data were fit, and the results are shown in Table I. Each entry lists the  $\chi^2$  of the fit and the fitted value of  $f_c^2 (\times 1000)$ . The first entry is the 1997 Nijmegen  $np$  PWA result [30]. Simply adding  $V_{\pi\gamma}$  increases  $\chi^2$  by a factor of 8. This can be greatly reduced by refitting only  $f_c^2$ , which is shown in the interim (next) entry. Refitting both  $f_c^2$  and the phenomenological interior region ( $r < 1.4$  fm) leads to the rightmost entry. Only those entries labelled “all” should be compared.

Although it is necessary to increase slightly the strength of the tail of OPEP to compensate for the addition of  $V_{\pi\gamma}$  in the overall fit, there is negligible change in  $f_c^2$ , and only a tiny decrease in  $\chi^2$ . The  $np$   $^1S_0$  scattering length and effective range also show negligible change. We conclude that the addition of  $V_{\pi\gamma}$  has not affected any of the former conclusions of the Nijmegen PWA with regards to  $f_c^2$ , quality of fit, or low-energy scattering observables. We emphasize that this does not mean that  $V_{\pi\gamma}$  is everywhere negligible, since that part with  $r < 1.4$  fm is subsumed in the phenomenological interior region of the Nijmegen PWA. If a force were to be constructed (for all  $r$ ) with an explicit  $\pi\gamma$  component, the effect of the latter would be considerably larger. This  $np$  force is repulsive in S-waves for  $r < 3.7$  fm, but attractive otherwise. A rough estimate using *ad hoc* form factors and several  $pp$  potentials shows that the *magnitude* of the  $np$   $^1S_0$  scattering length decreases by  $\sim 0.67$  fm. This is twice the strength of the effect found earlier from the double-seagull mechanism[27]. The deuteron energy is raised by roughly 60 keV.

As a check of our procedures, we have also fitted the cases  $\bar{\gamma} = \gamma_E \pm 1$ . These alternatives to our renormalization scheme either add or remove a specified fraction ( $2\alpha/\pi$ ) of OPEP from  $V_{\pi\gamma}$ , and this must be compensated in the final fit by a corresponding change in  $f_c^2$ . This is indeed found. Choosing  $\bar{\gamma} = 0$  (no finite renormalization) lowers  $1000f_c^2$  by 0.18, which is slightly more than half the quoted uncertainty. We elect, however, to use our

preferred convention ( $\bar{\gamma} = \gamma_E$ ), since that leaves  $f_c^2$  essentially unchanged.

In summary, we have calculated 19 Feynman graphs that produce the leading-order one-loop EM corrections to OPEP. The results are gauge invariant and remarkably simple, and generate a static charge-dependent  $\pi$ - $\gamma$ -exchange potential. This potential, which has a nominal strength  $(\alpha/\pi)V_\pi$ , was one of the few isospin-violating mechanisms not previously incorporated into the tail of the NN force by the Nijmegen PWA. When incorporated using our renormalization prescription, there is negligible change in the  $^1S_0$  low-energy parameters and the  $\pi^\pm NN$  coupling constant,  $f_c^2$ , and only a tiny improvement in the quality of fit.

The work of UvK, JLF and TG was performed under the auspices of the United States Department of Energy. MR acknowledges the financial support of the Australian Research Council. We would like to thank S. -N. Yang for several useful conversations. One of us (UvK) would like to thank G. A. Miller, R. B. Wiringa, and E. M. Henley for several useful discussions, while JLF acknowledges the help of S. A. Coon.

## REFERENCES

- [1] E. M. Henley, in *Isospin in Nuclear Physics*, D. H. Wilkinson, ed. (North-Holland, Amsterdam, 1969), 15; E. M. Henley and G. A. Miller, in *Mesons and Nuclei*, M. Rho and G. E. Brown, eds. (North-Holland, Amsterdam, 1979), Vol. I, p. 405.
- [2] G. A. Miller, B. M. K. Nefkens, and I. Šlaus, Phys. Rep. **194**, 1 (1990); G. A. Miller and W. T. H. van Oers, in *Symmetries and Fundamental Interactions in Nuclei*, W. Haxton and E. M. Henley, eds. (World Scientific, Singapore, 1995).
- [3] Y. Wu, S. Ishikawa, and T. Sasakawa, Phys. Rev. Lett. **64**, 1875(1990); erratum **66**, 242 (1991).
- [4] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C **51**, 38 (1995); B. S. Pudliner, V. R. Pandharipande, J. Carlson and R. B. Wiringa, Phys. Rev. Lett. **74**, 4396 (1995); R. B. Wiringa (private communication).
- [5] U. van Kolck, Ph. D. Thesis, University of Texas, 1993; U. van Kolck, Few-Body Systems Suppl. **9**, 444 (1995).
- [6] S. Weinberg, Physica **96A**, 327 (1979); S. Weinberg, Nucl. Phys. **B363**, 3 (1991); Phys. Lett. **B251**, 288 (1990); Phys. Lett. **B295**, 114 (1992).
- [7] S. Weinberg, Trans. N. Y. Acad. Sci. **38**, 185 (1977).
- [8] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984); H. Georgi, Phys. Lett. **B298**, 187 (1993).
- [9] J. L. Friar, Few-Body Systems (in press).
- [10] U. van Kolck, J. L. Friar, and T. Goldman, Phys. Lett. **B 371**, 169 (1996).
- [11] J. R. Bergervoet, P. C. van Campen, R. A. M. Klomp, J-L. de Kok, T. A. Rijken, V. G. J. Stoks, and J. J. de Swart, Phys. Rev. C **41**, 1435 (1990); V. G. J. Stoks, R. A. M. Klomp, M. C. M. Rentmeester, and J. J. de Swart, Phys. Rev. C **48**, 792 (1993).
- [12] O. Schori, et al. , Phys. Rev. C **35**, 2252 (1987); B. Gabioud, et al. , Nucl. Phys. **A420**, 496 (1984).
- [13] J. R. Bergervoet, P. C. van Campen, T. A. Rijken, and J. J. de Swart, Phys. Rev. Lett. **59**, 2255 (1987); R. A. M. Klomp, V. G. J. Stoks, and J. J. de Swart, Phys. Rev. C **44**, R1258 (1991); V. Stoks, R. Timmermans, and J. J. de Swart, Phys. Rev. C **47**, 512 (1993).
- [14] M. L. Goldberger and S. B. Treiman, Phys. Rev. **110**, 1178 (1958); Y. Nambu, Phys. Rev. Lett. **4**, 380 (1960).
- [15] G. J. M. Austen and J. J. de Swart, Phys. Rev. Lett. **50**, 2039 (1983).
- [16] V. Bernard, N. Kaiser, and U.-G. Meißner, Int. J. Mod. Phys. **E4**, 193 (1995) is a comprehensive current review.
- [17] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics*, (McGraw-Hill, New York, 1964). We use the metric and conventions of this reference. Our pion EM interaction, however, is written in isospin notation. Our proton charge,  $e$  (corresponding to the minimal substitution  $[p^\mu - eA^\mu]$ ) is denoted by  $e_p$  in that reference.
- [18] M. N. Butler, M. J. Savage, and R. P. Springer, Nucl. Phys. **B399**, 69 (1993). See Section 3 of this work for their renormalization scheme.
- [19] D. O. Riska and Y. H. Chu, Nucl. Phys. **A235**, 499 (1974). This CSB calculation was primarily concerned with isobar contributions, which we have ignored.
- [20] S. Furui, Nucl. Phys. **A254**, 370 (1975). This CSB calculation examined the effect of isobars.



- [21] M. Weyrauch and A. W. Thomas, University of Adelaide preprint, 1989 (unpublished). The CSB effect of crossed-box diagrams was considered.
- [22] L. K. Morrison, *Ann. Phys. (N. Y.)* **50**, 6 (1968). Only vertex graphs were considered.
- [23] J. S. Leung and Y. Nogami, *Nucl. Phys.* **B7**, 527 (1968). Both CD and CSB effects (static and nonstatic) were considered in this Coulomb-gauge calculation that included isobar box graphs and seagull contributions.
- [24] M. K. Banerjee, Univ. of Maryland preprint 75-050, 1975 (unpublished); T. B. Wells and M. K. Banerjee, *Bull. Am. Phys. Soc.* **20**, 1193 (1975). Only the box and crossed-box diagrams were calculated.
- [25] M. Chemtob, in *Interaction Studies in Nuclei*, ed. by H. Jochim and B. Ziegler (North-Holland, Amsterdam, 1975) p. 487. All diagrams were discussed but some were not calculated. Isobar contributions were included in this Feynman gauge calculation. The dominant isospin-violating effects were CD. The results are primarily in numerical form. See, M. Chemtob and S. - N. Yang, *Nucl. Phys.* **A420**, 461 (1984), and S. - N. Yang and P. U. Sauer, in *Few-Body Problems in Physics*, Vol. II, B. Zeitnitz, ed. (Elsevier, Amsterdam, 1984), p. 111.
- [26] T. B. Wells, dissertation, Univ. of Maryland, 1978 (unpublished); T. B. Wells, *Bull. Am. Phys. Soc.* **22**, 616 (1978). Box and crossed-box diagrams containing isobars were calculated.
- [27] J. L. Friar and S. A. Coon, *Phys. Rev. C* **53**, 588 (1996). The CD effect of box, crossed-box, and double-seagull graphs was calculated in Coulomb gauge.
- [28] S. N. Yang (private communication) pointed out to us that the potential of Ref. [27] has the opposite sign from some previous results. In the static limit the energy shift corresponding to Fig. (1g) can be written using old-fashioned (indeed, ordinary second-order) perturbation theory, which reproduces Eq. (6) of Ref. [27]. Our present calculation agrees with that sign.
- [29] J. L. Friar, *Ann. Phys. (N. Y.)* **96**, 158 (1976).
- [30] R. A. M. Klomp, J.-L. de Kok, M. C. M. Rentmeester, T. A. Rijken, and J. J. de Swart, Contribution to the 14th Few Body Conference, May 26-31, 1994, Williamsburg, VA, U.S.A., p. 95; M. C. M. Rentmeester *et al.*, (in preparation).



TABLES

Fit	$V_\pi$ [all]	$V_\pi + V_{\pi\gamma} [f_c^2]$	$V_\pi + V_{\pi\gamma}$ [all]
$10^3 f_c^2$	74.96(34)	75.22	74.98(33)
$\chi^2$	4223.6	4236.5	4222.8

TABLE I. Nijmegen PWA fits. The  $\pi^\pm NN$  coupling constant,  $f_c^2(\times 1000)$ , and the corresponding  $\chi^2$  of fit for a variety of cases are indicated in the top row. The potentials are those included in the tail of the  $np$  force, while the quantities in brackets were those fit to produce the results listed directly below. “All” denotes that the parameterized interior region of the force was fit in addition to  $f_c^2$ .

FIGURES

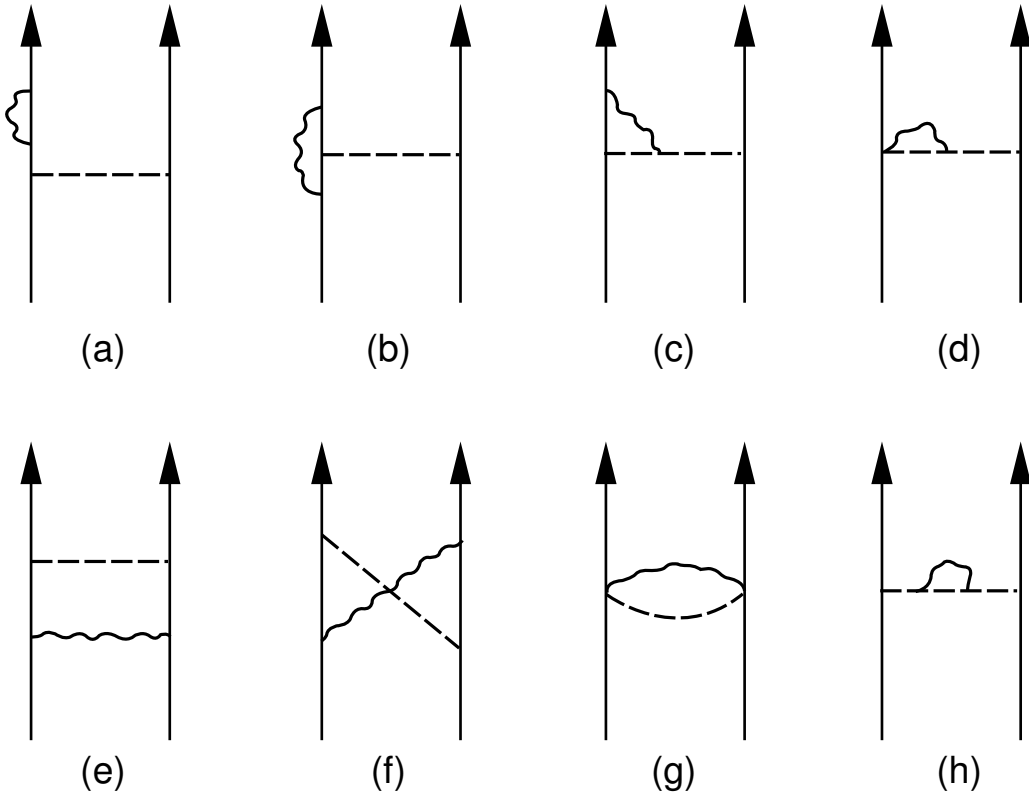


FIG. 1. Electromagnetic loops contributing to isospin violation in the OPE nuclear force in leading order. Solid lines are nucleons, dashed lines are pions, and wavy lines are photons.